

# THE LABOUR THEORY OF VALUE, AND THE DOUBLE TRANSFORMATION PROBLEM

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## Introduction

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**Abstract.**- If there is a “transformation” from direct prices ( $w$ ) into production prices ( $p$ ), it should be a second one from production prices into market prices ( $m$ ). Market prices are the great oblivion of the whole literature on the labour theory of value (LTV) and the Transformation Problem (TP). When these prices are taken seriously into consideration, one needs to deal with three equations of values-prices, not two, and market prices must appear in the right side of all of them. As a consequence, the conventional expressions of values and prices become wrong included those of the New Interpretation (NI) and the Temporal Single System (TSS). As a further result, all Marxian totals—the total value of the output and the total surplus-value, among others—remain invariable through both transformations, and there is just one single rate of surplus-value and one single rate of profit. The “temporal” equations used by the TSS are also misguided since time should be treated in another way according to which the static equations are compatible with a dynamic approach. Lastly, a summarized history of the historical debate on this issue helps to see that this approach, as the most general one, can shelter as particular cases the prevailing views.

## INTRODUCTION

Although this paper is also concerned with more general issues of the labour theory of value—in fact, what is presented here is intended to be a new, more general and *dynamic* approach to the LTV and the Transformation problem that fits better for the *complete* (not partial) framework required for the analysis of capitalist societies—one crucial aspect, the most visible one, regards the necessity of a new set of equations of values and prices that illustrate the terms of the transformation process. In this regard, it is my first contention that in order to deal properly with what should be called the “double transformation” problem<sup>i</sup>, we need to start from *three* equations, not two. If we call the vectors of direct prices  $w$ , production prices  $p$ , market prices  $m$ , we should write:

$$\begin{aligned} (1) \quad w &= m (A + B + B\rho) \\ (2) \quad p &= m (A + B + Kr) \\ (3) \quad m &= m (A + B + KR) \end{aligned}$$

[where  $A$ ,  $B$  and  $K$  are respectively the conventional matrices of material inputs (included depreciation), workers consumptions and stocks of (fixed and circulating) capital, all per unit of output;  $R$  is the diagonal matrix of actual sectoral rates of profit;  $\rho$  the rate of surplus-value, and  $r$  the general or average rate of profit].

In what follows, before and after explaining in more detail those equations<sup>ii</sup>, we will deal with other issues. We begin in this introduction by making explicit the main features of our conception of value. In the first section, we treat the question of the physical variables versus labour-and-monetary variables as determinants of value and price. In the second section we focus on a Table that shows the set of different relationships between the different categories of value and price that should be taken into account for a proper understanding of the problem. Section III deals with the reformulation of the equations needed to pose the problem in its correct terms. Section IV goes deeply into the process that produces values and prices: labour,

and tries to bring to light the different treatment that should be accorded to direct and indirect labour in this respect. Section V deals with time and puts forward a dynamic approach different from the TSS approach. And section VI compares the history of the debate about the TP with the new setting proposed in the paper, intended to be more general. Finally, after drawing the main conclusions, an appendix goes into further details on the issue of the evaluation of constant capital.

We begin by enumerating the main features of this approach (1 to 9). Some of them are of course shared by many other authors, but some others will appear for sure quite new and surprising. However, from the point of view of this paper, the long history of the debates concerning this issue makes clear that all previous interpretations show some kind of limitations that would to a great extent be overcome had it be used equations (1)-(3). These limitations include:

- a. All participants in the debate forget equation (3), and to our knowledge nobody<sup>iii</sup> employs  $m$  in the definition of the right side of the equations.
- b. All authors involved in the debate share a different version of equation (2) where  $m$  is always replaced by  $p$  in the right side.
- c. Roughly speaking, there have been three different and successive sets of precedent versions of equation (1), depending on what kind of prices are used in its right side. The Bortkiewicz tradition, imitated by many Marxists, multiplies the parenthesis by  $w$ ; the New Interpretation multiplies  $A$  by  $w$ , but at the same time  $B$  and  $K$  by  $p$ ; and lastly a minority of authors to whom we will refer later multiply the parenthesis by  $p$ .

To overcome these limitations we need a different setting of the problem beginning with a conception of the value question that is to a great extent new. The main features of the new dynamic-and-extended approach to values and prices can be presented as follows:

1. The value of a commodity is the quantity of human labour needed to reproduce at present one unit of it in the prevailing (normal) technical and social conditions. However, values are necessarily expressed in money terms. Quantities of value are therefore *measured* in quantities of money (for instance, euro), not in quantities of time, because it is only money what can express the quantities of social labour. As Marx puts it, “money as a measure of value, is the phenomenal form that must of necessity be assumed by that measure of value which is immanent in commodities, labour-time” (Marx, 1867, p. 51). Therefore, we agree with him in that price is just “the money-name of the labour realised in a commodity”, even if it is also true that “the name of a thing is something distinct from the qualities of that thing” (ibid., pp. 53-4).
2. The set of “techniques” used in production does condition the process of production of new value and surplus-value, but it is crucial to understand that those techniques are at the same time socially and economically determined, and more specifically they are determined at their turn by *value* relations (which as deduced from point 1 must be interpreted as both labour and money relations). The main mistake of most critics of the LTV consists of thinking that, when analyzing prices, the so-called “physical relations” can be conceived of as the independent variable determining, alongside with the distributive relationships, value and price (see Steedman, 1977). This erroneous idea is unfortunately shared by some supporters of the LTV. However, it seems to us that just the opposite is true: it is value relations (let us insist: both labour and money relations) what determines the use and the role of the different techniques and physical relations of production in a capitalist context, alongside with the technical relations themselves.
3. It is very often forgotten that the main purpose of any theory of value is to explain the behaviour of actual—real—market<sup>iv</sup> prices, as different from production prices. It is crucial to our interpretation to consider market prices ( $m$  in our equations) as the point of departure of the whole analysis, as they are the real phenomena that has to be explained as the primary expression and result of real labour, the main and deepest fact and concept of the LTV. By saying this we are not forgetting that the LTV is about labour, but it should be added that it is

more precisely about how labour explains (among other things) market prices. Many participants in the debate about the LTV and the TP—we would say an overwhelming majority—seem not to realize that the magnitudes that they and everybody are using in their analysis (as direct and production prices of commodities or their labour-value counterparts) are only concepts, not facts. We are fully aware that value relations (social labour relations and social price relations as regulators of market prices) are *completely and fully real phenomena*, but what are more often called “values” (our  $w$  in their monetary expression) and “production prices” (our  $p$ ) are theoretical concepts devised to dealing with the sole real prices,  $m$ , which should not be conflated with none of the former.

4. Partly due to this oblivion, almost the entire list of participants in the debate have been dealing with values and prices that are *similar* to Marx’s, at most special cases of Marx’s, but not exactly Marx’s<sup>v</sup>, which has resulted in numerous misunderstandings. This practice has been an enormous mistake because Marx’s values and prices are the best and more general instruments we have at hand, at least up to now, for the understanding of real prices. Therefore, the values and (production) prices used by most authors—that could be called “Bortkiewicz<sup>vi</sup> prices” (Rodríguez-Herrera, 1994)—are not the correct ones and do not express adequately Marx’s thinking<sup>vii</sup>. However, these “canonical” prices and values, the prevailing equations known by everybody, may have a double utility. On the one hand, as will be shown in the last section, they can be used as imperfect or intermediate or incomplete approximations of the correct and more general Marxian values and prices. But on the other hand, they can be used as a remainder of what means the Spanish phrase: “Por la boca muere el pez”.

5. The use of this approach makes possible to write a set of equations for all, direct, production and market prices, which allows us to proof that all the so-called “Marx’s equalities” hold simultaneously true. Contrarily to most authors, supporters<sup>viii</sup> as well as critics of the LTV, who consider this impossible, our equations prove that *for the economy as a whole* the stock of capital, the costs, the prices, the surplus-value as well as the rates of profit and surplus-value, all maintain their magnitude unaltered through the process of (double) “transformation”.

6. The author is convinced that all the work already made in order to develop empirical measures of any labour-value categories, including the work specifically directed towards the calculation of statistical correlations and other links between market prices and labour values (think of the work of Shaikh 1984, Ochoa 1989, and his followers) keeps being of an enormous importance for the LTV<sup>ix</sup>, even if many of the criticisms we have done above can be applied to those authors too. However, one of the unintentional, collateral results that can be reached from our equations is the foreseeable improvement in the quantitative approximation to actual prices due to the fact that in them the market prices of inputs have replaced their direct or production prices. This would be particularly observable for all statistical measures that start from input-output data, but could be used as well in other, more unexplored fields<sup>x</sup>.

7. One of the main features of this approach is its “dynamic” point of view. In this respect, the TSS contribution has been very helpful in improving the quantity and quality of the whole debate. However the way in which TSS treats time seems inappropriate and may induce a misguided setting of the problem. According to our own interpretation, these authors have been close to the proper treatment but have at some point deviated from the correct intuition, perhaps due to the obstacle posed everywhere by the problematic use of time that characterizes economics. We believe it necessary to distinguish between two different dimensions of time. One is logical time, the other is real time, and both are equally necessary to understand the question of value. It is true that the oblivion on the part of many authors of real time is a great limitation, but note that when other authors share their mathematical tools—not their assumptions—they are not necessarily setting the problem in non-dynamic terms. Therefore this setting does not deserve the generalized condemn made by the TSS and other authors of the use of comparative-static methods in any circumstance, even in framework compatible with a dynamic approach. This aspect of the TSS represents in our opinion a dangerous and wrong turn<sup>xi</sup>.

8. The description of the problem that has been already made helps us to understand why most authors, unconsciously feeling themselves defeated by the supposedly overwhelming arguments of the critics of the LTV, have perhaps forgotten that Marx did not limit himself, as

the Classics did, to postulate the LTV. As shown in Guerrero (2006), Marx made a double *demonstration* of the LTV, and this oblivion on the part of most supporters of the LTV might perhaps be explained mostly by what can be called the “psychological weakness of the defeated”. Undoubtedly this has also contributed as a factor in the path followed by the historical debate up to now.

9. Although we will not develop here other consequences of adopting the view proposed in this paper, it is clear that they can be hardly exaggerated. Let us cite as an example the necessary revision it would imply of crucial aspects of another debate: the debate about the possibilities of economic calculation under socialism. Most authors seem to interpret the defects of the Bortkiewicz conception of values and prices as defects of the Marxian LTV. When some authors in this specific debate defend the need to modify “labour-values” by *adding* the rent of land and other natural resources in the computation of the costs of production, or the need to charge an interest over the resources “stored” in time, they are typically considering the lack of doing so as an error of the LTV, erroneously conceived of as a “pure” labour theory (see Dickinson, 1939). Therefore it is no surprise that they either propose to correct the mistake for a proper calculation under socialism or avoid the problem by assuming prices proportional to labour contents (Cockshott and Cottrell, 1995, pp. 53-54, 110). But as our equations make evident, ground-rent and interests are in fact included in the prices formed according with the LTV (see the appendix), and therefore those who believe it necessary to use those prices or values as a guide under socialism do not need to make any addition to them at all.

## I. THE PHYSICAL AND THE MONETARY

Some ideas of Marx are so clearly expressed that almost none will deny they belong to him, either they share them or not. One of those ideas is that profits are the monetary expression of surplus-value whose material origin is the mass of surplus-labour produced by all wage workers and appropriated by the capitalist class. Therefore, the LTV states that, contrarily to what some seem to think, the *capitalist rate of profit as a real fact* (actual profits produced divided by actual capital) can neither exist nor be conceived of nor be calculated without the existence of *actual processes of labour*, and therefore without the existence of real labour relations that make possible the production and circulation of values.

This is so not only because without labour there simply can be neither production nor techniques of production to be used in it, nor any class of physical or monetary quantities at all as described by input-output matrices—which are always, let us remember it, the inputs and outputs of  $n$  labour-production (production = labour) process—but also because of two additional facts. On the one hand, because, as we will see and contrarily to the prevailing use, the vector of the *exact* production prices cannot be calculated as an eigenvector, but have to be derived from labour and monetary quantities. And in the second place and above all because, as has already been said, the so-called physical data (and matrices) are not independent of the set of prices and values that co-determine them, so that it is instead the value-surplus-value relations—that again imply both labour and money<sup>xii</sup>—what constitutes the true primary data of the problem and determines all other variables, included the so-called “physical” ones<sup>xiii</sup>.

Let us take, as an example of such “physical” matrices that are in fact not truly physical (at least not merely physical), the Leontief matrix  $A$  of intermediate or input-output coefficients (we suppose that their  $a_{ij}$  already include depreciation charges on the fixed capital advanced). If we present Marx’s ideas in accordance with the input-output language, we must start from recognizing that Leontief’s original idea was also that  $A$  depends on price data<sup>xiv</sup>, so that the entire input-output matrix must be better conceived of as the result of pre-multiplying the matrix of “constant capital” (in money, or money per unit of output when defined in unit terms),  $C$ , by the inverse of the (diagonal) matrix of market prices,  $M^{-1}$ :

$$(4) \quad A = M^{-1} C$$

[where the reciprocal of prices,  $1/m_1, 1/m_2, \dots, 1/m_n$  are the elements in the main diagonal of  $M^{-1}$ ; each  $c_{ij}$  element of matrix  $C$  is the sum of money that sector  $j$  spends in the purchase of the inputs coming from sector  $i$ ; and therefore the elements of  $A$  result to be  $c_{ij}/m_i$ . As for the units

used, as the  $(1/m_i)$  are measured in units of commodity  $i$  per euro, and the  $c_{ij}$  in euro per unit of  $j$ , the resulting  $a_{ij}$  are measured in units of  $i$  per unit of  $j$ ].

Accordingly, if we retrace our steps by multiplying the vector of market prices,  $m$ , by matrix  $A$ , we obtain the vector ( $c$ ) of total constant capital spent in the year by each sector (always per unit of output):

$$(5) \quad mA = m M^{-1} C = i \cdot C = c$$

[where  $i$  is here the unit vector].

Exactly the same can be done for matrix  $B$  (the physical components of the basket of commodities actually used as unproductive consumption by the wage-workers<sup>xv</sup> for their reproduction as wage-workers<sup>xvi</sup>) that can be defined as:

$$(6) \quad B = M^{-1} V, \quad \text{so that}$$

$$(7) \quad mB = mM^{-1} V = iV = v,$$

[where  $V$  is the matrix and  $v$  is the vector of the variable capital spent by each branch].

Therefore, the vector of market prices is not but the sum of the constant and variable capital spent in production plus a profit which amounts to a certain percentage of the invested capital. As we will see later, these prices can be interpreted theoretically as the result of a double process of transformation-distribution of the surplus-labour included as a component in direct prices, giving place successively to prices of production and market prices. Thus we write the latter:

$$(8) \quad m = m(A+B) + mKR$$

$$= c + v + mKR$$

$$= c' + mKR$$

$$= c' + s''$$

[i.e., market prices are equal to the sum of the “cost-price” ( $c'$  = the sum of the annual flows of both constant and variable capital =  $c + v$ ) plus an individual profit ( $s''$ ) or a percentage of the entire capital invested in each individual branch, i.e. equal to  $mKR$ , where  $K$  is the matrix of stocks of capital<sup>xvii</sup> per unit of output and  $R$  is the diagonal matrix of actual rates of profit in each sector].

## II. The case for an enlarged set of “values” and “prices”

But let us make a step forward and turn from “physicalism” to other crucial aspects of the LTV, thus enlarging the relationship between values and prices towards a more general view. We believe that in order to properly understand the various dimensions needed for a *general* analysis of value it is as well necessary an enlarged set of (more) unambiguous concepts or dimensions or meanings of values and prices. In our view, many of the problems of interpretation that characterize the historical debate are due to the use of an insufficient number of categories. Table 1 intends to amend this by collecting the complete set of categories needed for a general explanation of prices starting from Marx’s theory. This is made according to what we interpret to be the spirit of *Capital* (even if not necessarily using the same terms), but it should be reminded that, whether the table represents Marx’s ideas or not, we still think it necessary to use at least all the categories shown in it for a correct understanding.

A Values ( $\alpha$ , in time)	B Exchange Values		
	B <sub>1</sub> Relati ve	B <sub>2</sub> Gold	B <sub>3</sub> Prices ( $\beta$ , in contemporary,

			value s (i/j) = Relati ve prices (i/j)	prices (g)	credit money)
C 'Theoretical' (ideal) values and prices	I	1. Individual value $i_H$	$i_{ij} =$ $i_{Hi}/ i_{Hj}$	$i_{ig} =$ $i_{Hi}/ i_{Hg}$	1'. Individual price $i = i_{Hi} * \pi$
	II	2. Direct value $w_H$	$w_{ij} =$ $w_{Hi}/$ $w_{Hj}$	$w_{ig} =$ $w_{Hi}/$ $w_{Hg}$	2'. Direct price $w = w_{Hi} * \pi$
	III	3. Production value $p_H$	$p_{ij} =$ $p_{Hi}/$ $p_{Hj}$	$p_{ig} =$ $p_{Hi}/$ $p_{Hg}$	3'. Production price $p = p_{Hi} * \pi$
D Real (actual) values	IV	4. Market value $m_H$	$m_{ij} =$ $m_{Hi}/$ $m_{Hj}$	$m_{ig} =$ $m_{Hi}/$ $m_{Hg}$	4'. Market price $m = m_{Hi} * \pi$

Table 1: The complete set of categories of value and price, according to our own interpretation of Marx's LTV.

Looking at the table, we must first of all say that the “transformation” has to imply the passing from some place in this Table to another one, i.e. it needs to consist of a certain kind of “movement” inside the table. However, it is important to note that this movement or movements must be almost always understood only as *mental processes*, except those going from market prices to market values and vice versa (the row IV) since they are the results of actual material processes taking place in real life. In his critique of Itoh (1992), Shaikh reveals a similar idea when he affirms that Itoh and he agree in that “prices of production never exist as such”<sup>xviii</sup>.

Secondly, we must emphasize the necessity of distinguishing the different nature of two kinds of relevant movements: “horizontal” movements on the one hand (those going from part A to part B of the Table, *and vice versa*) and “vertical” movements on the other (both upward and downward from C to D, *and vice versa*). Of course, this distinction contrasts with the prevailing, non-Marxist (but also Marxist) interpretation of the critics of the LTV who prefer to think that the TP consists of simply and directly travelling from 2 to 3'. In this treatment, no distinction or decomposition is made between horizontal and vertical movements, and this almost universal conflation, that amounts to conceiving the trip as a passage from what is called “the Marxian world” to “the market world”, has generated countless misunderstandings.

Now, the relationship between A and B expresses the necessary relation between the inner magnitudes of values and their monetary expression as exchange values. As true values only exist when all products are produced as commodities, and this implies the existence and functioning of a general equivalent of all other commodities, values have to be expressed in certain quantities of the general equivalent, money<sup>xix</sup>. Therefore, all variables in the A side of the table are *expressed* in hours per unit of commodity, and those of the B side are *measured* in euro per unit of commodity. But as money is and acts as a special commodity because it is the only general *equivalent* of all other commodities, it is clear that when any singular commodity is actually being related to money in the market it is in fact being related (as values) through the market with all other commodities (as values).

Therefore, converting values from labour into money is thus just to *compare* each type of commodity (and its content) with the rest of them (and their content). It is just to interpret the value of each commodity in a comparative or *relative* way, either it is compared with *any* other

specific commodity (as commodities  $i$  and  $j$  in column  $B_1$  of the table) or with the “average” or “social” commodity, only susceptible of being represented by money—either gold money (column  $B_2$ ) or credit money (column  $B_3$ ).

However, linking a *specific quantity* of labour-value necessary for reproducing one unit of commodity  $i$  with the exact specific quantity of money *representing the same basic fact*—the need to spend a *certain* quantity or fraction out of the mass of social labour for reproducing one unit of  $i$ —requires something more. It requires in addition conceiving of and measuring value as a certain quantity not of one or another type of labour but of “average”, “social” labour (simple human labour). Thus, dividing any price in the right side of the table, any “B-price” as can be called, by *the average, social productivity of labour in terms of money*—what we call  $\pi$  and is equivalent to the “monetary expression of labour-time”, but preferable to the “inverse of the value of money”—, amounts to measuring the values, or “A-values”, in hours of average, social, simple, abstract labour, which is the only sort of labour that can appear in Table 1.

As a consequence, and this is an important conclusion, *all* B-prices in the table ( $i, w, p, m$ ) can be interpreted as the simple result of multiplying the A-values ( $i_H, w_H, p_H, m_H$ ) by  $\pi$ . And likewise, the A-values are just the result of dividing B-prices by  $\pi$ .

It can be seen that our “definition” coincides as a practical result with the “monetary expression of value” (Duménil and Foley, 2006) or the inverse of what Fine, Lapavitsas and Saad-Filho (2004) calls the “labor expression of money”. However, it is preferable to use “the average, social productivity of labour in terms of money” than the inverse of the “value of money” often used in the literature (see for instance Foley, 1982), in order to taking into account what according to Marx is the “contradiction” between the “two values” of money. As Foley himself writes, “the value of money as defined here will not be equal to the labour value of the money commodity” (1982, p. 39). And Rodríguez-Herrera has emphasized the same idea by insisting on Marx’s idea of the “contradictory character of the form of value” and its application to money, what means that any producer selling his product for money appropriates on the one hand the “value embodied in the use value” of a certain quantity of money, and on the other hand “the value represented” in it (1994, p. 20)<sup>xx</sup>.

In any case, our defence of the *average social productivity of labour* is at least as much an “unambiguous method of measuring the monetary expression of labour” as that of the NI, and thus cannot be the object of the criticism made by Foley to the TSS definition of the monetary expression of value of not being “single” nor “consistent” (Foley, 2000, pp. 24, 33). Similarly this method avoids other criticisms made against the NI: “This calculation, based on the definition of the value of money simply as the value commanded by money in circulation, detaches both money and its value from the monetary and financial processes that link money to the general movement of capital accumulation” (Fine, Lapavitsas and Saad-Filho, 2004, p. 9).

As for the exact quantification of  $\pi$ , and having into account that total output holds invariable through both transformations (see below):

$$(9) \quad wx = px = mx,$$

we reach the result that  $\pi$  can be defined either in gross terms (what we call  $\pi_1$ ):

$$(10) \quad \begin{aligned} \pi_1 &= mx/lx \\ &= px/lx \\ &= wx/lx \end{aligned}$$

or alternatively in net terms<sup>xxi</sup> ( $\pi_2$ ):

$$(11) \quad \begin{aligned} \pi_2 &= m \cdot (I-A)x / lx \\ &= p \cdot (I-A)x / lx \\ &= w \cdot (I-A)x / lx \end{aligned}$$

Therefore if we call all the A-values simply  $\alpha$ , and all the B-prices  $\beta$ , we can express every horizontal movements going from A to B and vice versa<sup>xxii</sup> in Table 1 as done in equation (12),

whereupon we can conclude that this kind of movements are simply a sort of “translation” from one language to another<sup>xxiii</sup>, which can be checked in the apparent chaotic way of expression of Marx in *Capital*, that is not but the result of this double correspondence:

$$(12) \quad \beta = \alpha \cdot \pi;$$

(or:  $\alpha = \beta/\pi$ )

As for the vertical movements, we have to distinguish between, on the one hand, the relationship going from the lower part of the table, *C*, towards the upper part, *D*, and on the other hand the relationships between the different levels included in *D*. As said above, most participants in the TP debate seem to forget that real values and prices (both  $m_H$  and  $m$ ) are the main data and the only *factual* evidence that must be analysed and interpreted by any theory of value and any theoretician, either they prefer to use the LTV or any else. Therefore, before looking more closely into the double top-down movement going from  $w$  to  $p$  and from  $p$  to  $m$  (i.e. the “double transformation” that we mentioned above) we have to insist in that in the reality of facts there exist only the  $m$  prices<sup>xxiv</sup> and that, as already said, those prices are one of the unavoidable logical *prius* of the developing of the LTV.

Let us add that only a few authors point correctly to the necessity of distinguishing between actual prices and production prices, being the latter neither the real ones nor indeed, contrarily to what many think, the prices that the capitalists take into account for their decisions. We have already cited Shaikh<sup>xxv</sup> and Itoh in this regard, but as soon as in 1957 Seton wrote that production prices “are only the first approximation to actual *market prices*” (1957, p. 149), and in a similar vein Duménil and Foley (2006) affirm that their interpretation reveal some properties that “hold for any set of prices, not specifically prices of production (...) Prices of production are just *one case*”, and “the existence of non-reproducible resources and their rents; counteracting factors, such as monopoly, may also prevent equalization of profit rates”. As will be seen in the last section of this paper, this brings these authors closer than others to our “general” approach to the LTV.

Once the latter has been clarified, there is another, more peacefully accepted feature of the LTV: it uses two main types of theoretical values and prices for analysing actual values and prices. We can now turn to these, more specific, movements relating levels or rows II and III (in both columns, or boxes 2-3, and 2'-3') in Table 1, to which is usually and mistakenly reduced the debate about the Marxian transformation idea. Apart from the fact that the TP is “double” (i.e. it goes down until 4 and 4', not just to 3 and 3'), the passage from levels II to III is also “double” in another sense, since it happens in both columns at the same time, and thus goes from 2 to 3, and from 2' to 3' too. This amounts to saying that it is rather a question of moving from level II to level III<sup>xxvi</sup> (and from III to IV as well if we interpret transformation as a double passage).

#### IV. The need of new equations for a new interpretation

We said in the introduction that *all equalities* mentioned by Marx<sup>xxvii</sup> are preserved through the TP, and we will in addition see in this section that this happens twice, both when passing from level II to level III (the usual transformation or “transformation 1”, so to say) and also from III to IV (“transformation 2”). To our knowledge almost none of the participants in the debate, at least those using simultaneous equations, have reached as a result that all equalities are preserved<sup>xxviii</sup>. But it is obvious that, although for each single commodity (or sector) all quantities involved (costs, prices, surplus-value...) are in general different when considered before and after transformation(s), the opposite is true when analysed at the level of the whole economy, as Marx thought. At this macro level, the stock and flows of capital, the material and wage costs, the prices, the surplus-value as well as the rates of profit and surplus-value, all hold unaltered after completing both passages (from II to III, and from III to IV).

As said, all what is needed, when representing Marx’s ideas in terms of the modern input-output language—which does not alter his own conception of values and prices (included  $m$  prices)—is first of all to begin with a set of three equations instead of two. Secondly, by using  $m$  for evaluating the inputs in the three equations, instead of using  $p$  for the equation of prices of

production and  $w$  for the equation of values, we make the costs and the stocks equal in the three cases. Therefore, it is not true either that there are two different rates of profit, as Foley thinks<sup>xxxix</sup>, and as a further consequence of this view the following reveals false as well:

“The dual system framework poses the problem as proving mathematically the proportionality of profit and the wage bill measured in market prices to surplus and necessary labor time measured in embodied labor coefficients, and argues that the New Interpretation makes no contribution to this project. If one accepts the mathematical correctness of the dual-system demonstrations, as I do, then within that framework the question is completely settled: it is impossible in general to maintain the required proportionality, and neither the New Interpretation nor anything else is ever going to come to a different conclusion without committing a mathematical error. Accepting the mathematical validity of the analysis within the dual-system framework, however, does not require one to accept that this is the only, or the most relevant, way of posing the issue.” (Foley, 2000, p. 24)

As, beginning from the new view, one should not accept “the mathematical correctness of the dual-system demonstrations”, the question is completely settled but contrarily to what Foley thinks: it is *possible* in general to maintain the required proportionality between monetary value added and direct labour. Therefore, this allows us to reach a conclusion opposite to Foley’s “without committing a mathematical error”.

But the use of market prices in all our equations does not depend of the previous result, but follows from our general setting of the issue. According to this, the starting point of the analysis should be to consider what happens in the process of creation of new value and surplus-value. We can follow Marx literally here, whose approach has nothing to do with the idea that physical data (the inputs used directly in production and indirectly via the subsistence baskets) determine prices and profits<sup>xxx</sup>. Such a relation between use values and values cannot be even imagined inside the framework of the LTV. What Marx wants to explain is how all capitalists, although taking as given a certain set of data that they think (correctly) they cannot control by ourselves—beginning with market prices—do not remain placidly still, but try their best dealing with as many variables as possible, above all the ones they think can be controlled more easily. Therefore, they pay attention first of all to what happens inside their own firms, and thus manage with all means within their reach to get as much profits as they can.

Capitalists know that they must act and struggle for profits. And for this they have to operate simultaneously in two battlefields: that of production and exploitation and that of circulation and competition, i.e. “inside” and “outside” their companies. And this is precisely why, for showing how capitalists have to confront a double struggle against a double enemy, and for the first time in the history of the economic thought, Marx arrived to the idea that it is necessary to distinguish between different types of regulating values and prices:

1) on the one hand, since capitalists must fight against the working class—and they can do this as “brothers”—for extracting as much unpaid labour as possible<sup>xxxxi</sup>, it is needed a category of prices that directly show this origin and the dependence of prices on the fact of exploitation: these are  $w$  prices, which are equal to the sum of costs plus a charge on wages (or paid labour) extracted from workers as surplus-value or unpaid labour;

2) but on the other hand, as they have to compete, now as fierce rivals, with all other capitalists<sup>xxxii</sup>, there is the need of other prices too that reflect the main aspects of this competition<sup>xxxiii</sup>: a) the *inter-sectoral* side of competition with its resulting tendency towards the equalization of the rates of profit, which is captured in prices  $p$ ; and b) *intra-sectoral* competition, which gives way to “individual” values and prices (the  $i$  prices) and also, as it includes the analysis of the ground-rent, etc., leads back to market prices,  $m$ .

The “transformation” has thus to do with the relationship between these two battlefields, and in fact involves not only rows II and III of Table 1—nor the trio formed by I, II and III—but, as we have seen, it has to do really with all levels of prices, from I to IV. Of course, all these prices spring from labour, but the reader should not be impatient in reaching the level of labour because this level is deeper and has to be got after starting from the surface. The problem with

incorrect analytical efforts is not that they use, let us say “appearances” as well as “essences”, but that they put them in relation in the wrong way.

We have thus forgotten neither labour nor values, but are simply being consistent with our method of proceeding from market prices,  $m$ , in direction of labour. We propose, following Marx’s suggestions, to make the two steps described above, and thus use production and direct prices as intermediate steps towards the understanding of market prices. But the definition of all these prices has to take into account something that can be summarized in a simple manner:

- 1) each capitalist takes *market prices* as given;
- 2) but operates *actively* in the pursuit of a maximum individual quantity of *profits* out of those prices (past and present);
- 3) so that this search on the part of all of them makes the market prices change in time through the change of values, i.e. the quantities of labour and surplus-labour, created in production.

We have said that these prices,  $m$ , are part of the data that capitalists take as *given*<sup>xxxiv</sup> because they escape to their control, at both the individual and the collective level. It would not be reasonable for them to treat those—numerous, relatively slow and hardly controllable—*market processes* as they treat their *own* single individual production process, the process that occurs on their closest arena, under their own jurisdiction. It seems thus wiser to take input prices as given, at least *in the short term*, and focus instead on the *inner* process, which is always a labour ( $l$ ) process (of course labour of others).<sup>xxxv</sup>

As a consequence, this double *active* struggle against workers and competitors in the pursuit of surplus-labour and profits, generates at any moment—during any *period* of time, rather—a certain mass of profit which brings to its “producer”, when compared to the capital he invested previously, a *specific individual rate* of profit that will in general be different from the rates of all other firms and sectors. We could understand nothing of any theory of value and price if we limited ourselves to these “individual” prices by themselves, and this is why Marx and we and everybody have to go beyond them. Suppose all firms in every branch get the same (sectoral) rate of profit and thus form the same (sectoral) market price. This is not enough either, and this is why all economists, at least since the physiocrats and Smith on, affirm that these prices fluctuate around regulating prices or prices that operate as centres of gravity or physical attractors of the former.

Actual prices—above all once it has been put aside all non-market elements, like taxes or subsidies—fluctuate persistently around (although do not converge to) “production prices” ( $p$ ), which are prices equal to the specific costs of production of every commodity (sector) plus a profit rewarding each branch with *the same percentage of the capital invested* in it. Therefore, production prices can be interpreted as the “result” of a mere “redistribution” of the real overall mass of profit according to the rule of a *uniform rate of profit* for all branches, on the understanding that we interpret correctly this redistribution not as a real but as a *mental process*, even if competition is as real a phenomenon as it is the exploitation of labour.

A crucial aspect of this interpretation is that we do not need to change the measure of the cost-price for reaching as a result of the previous analysis a genuine *new category* of price:  $p$ , as different from  $m$ . The difference between both of them is the different way in which profits are computed. If instead of beginning from  $w$  to  $p$ , we start from  $m$  in order to define  $p$ , what has been said allows us to “define”  $m$  and  $p$  as follows<sup>xxxvi</sup>:

$$(13) \quad m = m(A+B+KR)$$

$$(14) \quad p = m(A+B+rK).$$

[where  $r$  is the general rate of profit, formed as an average from the multiplicity of individual rates of profit included in matrix  $R$ , and profits are measured either as the product of capital  $mK$  by one specific (sectoral) rate of profit  $r_i$  of matrix  $R$ , or the product of the same  $mK$  by the general average rate of profit  $r$ ].

Equations (13) and (14) make clear that even if the cost-prices are the same for all B-prices—equal to the product of market prices by the “physical inputs” (as interpreted above), i.e. equal to  $m \cdot (A+B)$ —the magnitude of each individual market price is in general different from that of the individual production price, inasmuch as the unit profit contained in them is different too. Therefore, although for every individual sector actual unit profits are in general different from the ones that would result by using the same average profit in all branches, in the economy as a whole they are of the same magnitude:

$$(15) \quad mKR \neq mKr$$

$$(16) \quad mKRx = mKrx$$

[where  $x$  is the column vector of sectoral outputs].

Now, it is essential to understand what the crucial point for Marx is, and the novelty of his analysis. Marx accepts the traditional idea of  $p$  as direct “regulators” of  $m$ <sup>xxxvii</sup> but thinks that this is not enough for understanding market prices. It is necessary to go beyond. Why? Because the  $p$  category leaves unanswered the main question of *why* all rates of profit—both the diverse individual rates of profits included in matrix  $R$  and the single social average rate of profit  $r$ —are *precisely what they are*, i.e. why they amount to a *certain quantitative level* and not to a different one. Definitively, Marx wants to know exactly the same thing that everybody interested in the theory of prices would and do want to know: *what* explains that the rates of profit included in  $p$  (and  $m$ ) prices are fixed at 20% (or gravitate around 20%), let us say, and not for instance at 0.2% or 2000%.

His answer to this question is well known. He says: because the overall sum of profits throughout the economy is just the monetary expression of a *certain* limited and perfectly *exact* mass of surplus-labour taken out as values from the working class as a whole. It is this exact quantity, no other. This is why, for correctly representing Marx’s conception of value and price, it is crucial to complement equations (13) and (14) with a third one corresponding to what can be called “direct prices” (our  $w$ ), i.e. the prices of commodities whose content in profits is proportional to the *fresh labour* put in motion by workers in their production—and therefore proportional to the fraction that represents their content in wages or variable capital too—which is what would occur *if* the equalization of sectoral rates of profit due to competition among capitals did not operate. In this case, direct prices ( $w$ ) would be the long-term direct regulators of market prices ( $m$ ).

After doing so, we finally get back to the trio<sup>xxxviii</sup> of equations advanced at the beginning of the paper (1 to 3) that summarizes our own view and at the same represent what, according to us, would be Marx’s position expressed in matrix algebra terms:

$$(1) \quad w = m (A+B+B\rho)$$

$$(2) \quad p = m (A+B+Kr)$$

$$(3) \quad m = m (A+B+KR)$$

[where  $\rho$  represents the rate of surplus value, as defined below (see section V)].

Note that, like in the previous reasoning, the cost-price per unit is the same for  $w$  as for  $p$  and  $m$ , both at the individual and the total levels—i.e. equal to  $m \cdot (A+B)$  and  $m \cdot (A+B)x$  respectively—whereas the profits, although identical at the overall level, are different when taken at the individual (sector) level:

$$(17) \quad mKr \neq mB\rho \neq mKR$$

$$(18) \quad mKrx = mB\rho x = mKRx$$

We can conclude that the purpose of this construction is to see how the exact amount of profits is determined from another exact amount: the amount of surplus-value produced. And therefore that the rate of profit is just a modified form of the rate of surplus-value, and ultimately that the magnitude of actual prices can only be understood as an expression of the actual quantity of labour that is needed for reproducing one commodity compared with the actual quantity of labour needed for reproducing other commodities. Therefore the three categories of prices are

the same *type* of expression of labour-values, but their exact quantitative definition depends exclusively on the quantitative definition of the surplus (labour and money surplus) above the costs.

We can now turn finally to labour.

#### V. Labour: direct and indirect. What is and how to measure indirect labour?

We must deal now with the relationship between our set of prices and particularly the prices we have called “direct prices” ( $w$ )—remember Marx: just another name for “direct” values ( $w_H$ )!—and what the literature equivalently calls “labour-values”. Both definitions agree in that labour-values are the “total amount of labour”—i.e. the sum of *direct* labour plus *indirect* labour—necessary to produce and reproduce a commodity at present in the prevailing or normal social terms. However both interpretations differ. Labour-values are usually represented as in equation (19), since most authors think that values are the sum of direct labour ( $l$ ) plus indirect labour as *computed* by the product of the matrix of inputs by *those same* “values” ( $\lambda A$ ):

$$(19) \quad \lambda = \lambda A + l$$

But values  $\lambda$  are not the correct ones even if, compared to “our” values (see equation (21)), they show the “advantage” of being interpretable as “coefficients of vertically-integrated labour” (see Pasinetti, 1973), as showed in equation (20):

$$(20) \quad \lambda = l(I-A)^{-1}$$

[where  $(I-A)^{-1}$  is the Leontief inverse that allows to convert direct labour into total labour].

We will depart again from the usual interpretation, because it is important to understand why even this traditional, simple and beautiful definition of values is incorrect (except in a limited and hypothetical world where any deviations of prices from values were impossible). The correct Marxian values are different (see equation 21) and this form of writing them, that makes most clear their origin in labour, cannot but coincide with the values of equation (1). For Marx and for us the constant capital involved in production is always the (constant) capital *actually*<sup>xxxix</sup> expended in the purchase of inputs, and this is so either we are “naming” those costs as the actually paid sum of *money*,  $m_A$ , or as the actual sum of *labour* it represents, in whose case this sum of labour can be calculated as the total price of inputs once converted into hours of simple labour, i.e.  $m_H A$  (where  $m_H = m/\pi$ , the market price of inputs divided by the average social productivity of labour).

Therefore our equation for values in labour terms is:

$$(21) \quad w_H = m_H A + l,$$

that once multiplied by  $\pi$  throughout, and thus translated into money terms, gives the equivalent:

$$(22) \quad w = m_A + L$$

[with  $L = l \cdot \pi$ ].

As already said,  $l$  is direct labour, the sum of (both paid and unpaid) labour put in motion by workers in production, and it can therefore be divided into two parts:

$$(23) \quad l = m_H B + s$$

[where  $s$  is surplus-labour in hours; and  $m_H B$  is the value of the labour-power, i.e. the equivalent in hours of the variable capital  $v$  or sum of money paid by the capitalists to their workers that allows them to buy at market prices,  $m$ , the entire mass of goods and services composing their actual consumption basket].

Therefore, *the rate of surplus-value*,  $\rho$ , can be equivalently defined as either a ratio of quantities of labour (surplus-labour divided by necessary labour) or a ratio of quantities of money, i.e.:

$$(24) \quad \rho = sx / m_H Bx \\ = s\pi x / mBx.$$

The same can be said of the *rate of profit* that can be defined in a double way too, but always as a scalar whose magnitude is uniquely determined:

$$(25) \quad r = sx / m_H Kx \\ = s\pi x / mKx$$

It is easy, by the way, to see how the rate of profit relates to the rate of surplus-value by means of the value composition of capital ( $q$ ):

$$(26) \quad r = sx / m_H Kx = \\ = (sx / m_H Bx) \cdot (m_H Bx / m_H Kx) \\ = \rho \cdot (mBx / mKx) \\ = \rho / q$$

[where  $q$ , the value composition of capital, is the ratio between the value of the stock of capital and the flow of variable capital spent in one year, both expressed either in labour terms or in money terms].

If we come back now to equations (1)-(3), we can conclude that the three kinds of prices—the B-prices of Table 1, that are not but another name for the A-values—only differ between them in their profit or surplus-value component<sup>xl</sup>. Therefore, if we call  $c'$  the cost-price, i.e. the sum of  $c + v$ , we can write:

$$(1') \quad w = c' + s' \\ (2') \quad p = c' + s'' \\ (3') \quad m = c' + s'''$$

[where profits, another name for surplus-values expressed in money, are defined threefold as they can be measured, as seen, by three different amounts:  $s' = mB\rho$ ,  $s'' = mKr$ , and  $s''' = mKR$ , respectively, all of them monetary expressions of  $s$ ; and of course,  $s' \neq s'' \neq s'''$ , but  $s'x = s''x = s'''x$ ].

It is true that this idea is not new in the literature on this topic (see for instance Moseley) but, in the state in which this aspect of the debate is at present, the authors often seem to be writing in quite different worlds. Our equations show that matrix algebra has no problem at all to make room for interpretations beyond Bortkiewicz's, Samuelson's or Sraffa's.

## VI. Time and technical change in the new dynamic approach

In Guerrero (1995), when arguing how much insightful Rubin's (1923) analysis of competition was compared to Sraffa's (1926), we showed that Sraffa's warning against a supposed tendency towards monopoly arisen from the existence of increasing returns of scale is a wrong view that can be explained as the result of an insufficient comprehension on his part of the meaning of time in the economic processes. Sraffa did not understand that the costs of the firm can be decreasing in time (see in Figure 1 the diminishing level of the successive optima of exploitation of the optimum scale of a firm at moments  $t_1, t_2, t_3, t_4\dots$ , due to the continuous improvement of techniques of production,  $T_1, T_2, T_3, T_4\dots$ ) and simultaneously increasing as a function of the quantity of output beyond the optimum scale of the firm<sup>xli</sup>.

In our opinion, two frequent and symmetric errors are typically made when dealing with this issue. For some authors, there is no question of time at all since they seem wiser to focus only

on the *scale* of the firm, which results in their well-known reduction to the comparative-static approach. This is the case of most Neoclassicals whose analytical framework does not need of any serious study of time since they habitually assume just one single and unchanged technique of production for all firms (the “representative firm”) in every sector. This is why they use the conventional handbook graphics for showing the relationship between quantities and costs (i.e. they look only at axes *x* and *y* in our Figure) and ignore the relationships between costs and time (axes *y* and *z* in Figure 1) or between the scale and time (axes *x* and *z*).

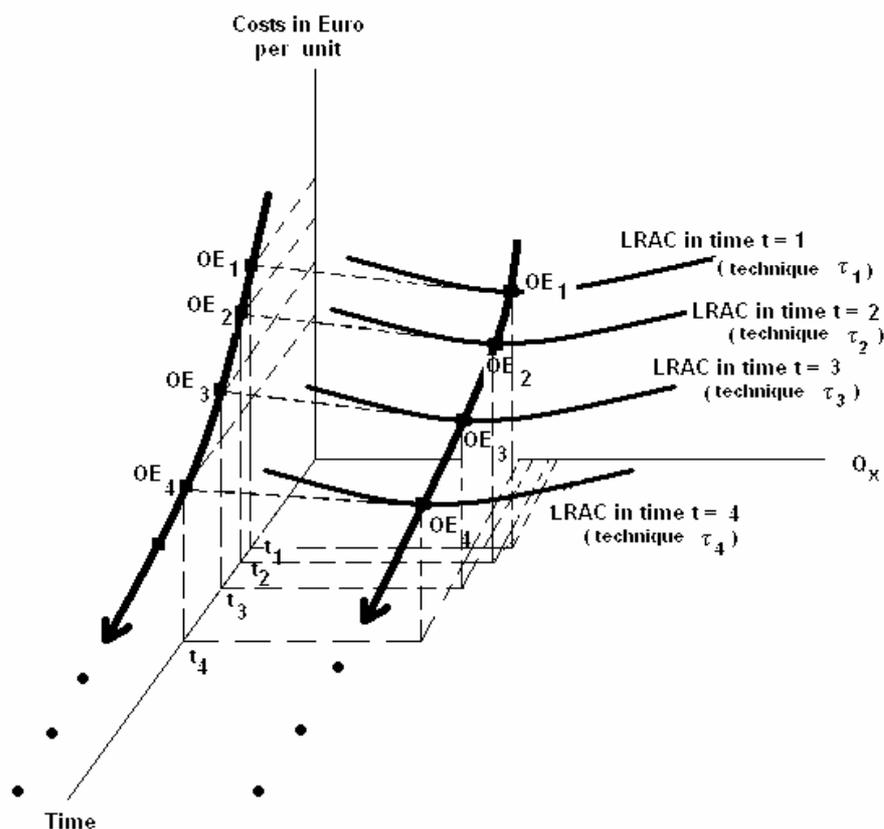


Figure 1: Minimum average costs of the firm in relation with both time and output

In contrast, the polar interpretation can lead to the opposite symmetrical error. It consists of thinking that it is not legitimate to use static-comparative methods because doing so amounts to ruling out the possibility of a dynamic perspective. It is as if in Figure 1 we could only use axes *y* and *z* but not *x* and *y*. This approach is not correct either, as a simile can make clear: one can think that it is preferable to register with a “video camera” what is happening to costs (or anything else) *along the time*, but still be interested in taking a photo of what is precisely the situation *at moment*  $t_1$ ,  $t_2$  or whatever. In fact we might need photos instead of videos in order to capture *some* aspects of reality that cannot be properly analyzed by means of a video.

Many supporters of the TSS seem to believe that the latter interpretation is not possible or is illegitimate, and thus condemn the use of simultaneous equations as if this framework were intrinsically incompatible with a dynamic approach. In our view they mistakenly translate some suggestions of Marx from the *logical* time—where they belong theoretically—into the real *chronological* time, what amounts to conflating the two improperly. No author outside the TSS seems to have realized this either.

In its pursuit of showing how values determine prices, the TSS has insisted correctly in the necessity of adopting a temporal framework of analysis. They emphasize that production is a process that takes place in time and therefore the value or price of the inputs, as they are

purchased at moment  $t$ , cannot be the same as the value or price of the outputs that have to be sold at moment  $t+1$ . In our opinion, this emphasis in the temporal dimension of production and value is a necessary and valuable step towards the correct understanding of the issue but the way in which the problem is posed by the TSS and the rejection of the tool of the simultaneous equations lead to a wrong solution.

Let us take as an example Ramos<sup>xiii</sup> (1998). He writes that for Marx the cost-price (that he calls  $K$ ) and the value ( $W$ ) are determined in different “phases” of the circuit of capital. When analyzing present values, the fact that the purchase of inputs has been made at prices of production that differed at that time from their values must be understood as a “past error” that has no consequence for the determination of present values. Ramos is correct in that the cost-price is a “premise” and has to be taken as a “given” fact in the process of production and circulation<sup>xiii</sup> that gives way to the determination of present values, but he thinks that “given” means “previously in time”, and not as in this paper, “logically previous”, i.e. a *prius* for the analysis. Therefore, he writes values and production prices in his temporal framework as follows:

$$(27) \quad K_t + P_{t,t+1} = W_{t+1}^{pt}$$

$$(28) \quad K_t + P'_{t+1} = P_{t+1}^{ct}$$

[where  $K$  is the cost-price,  $P$  is surplus value,  $P'$  is profit,  $W$  is value, the superscripts  $pt$  and  $ct$  mean “production time” and “circulation time” respectively, the subscripts refer to moments  $t$  or  $t+1$ , and the double subscript denotes the elapsing of time between  $t$  and  $t+1$ ].

What Ramos and the TSS want to show is that the prevailing “dual” approach is wrong in conceiving of values and (production) prices as if they were different and alternative “systems” or “worlds”. Of course these authors are right in that there are not two separate systems but just two parts of one “single”<sup>xliv</sup> system where prices and values are intertwined in the same manner in which the production process of capital and its circulation process belong to the same global process of reproduction of capital. But Ramos and the TSS think that, as all those processes take place in real time, it is the surplus value produced between moments  $t$  and  $t+1$  what constitutes the object and the amount to be distributed as profits at moment  $t+1$ , and similarly it is the set of prices formed at  $t+1$  what will constitute the future basis (or one of the bases) of the values that will be formed at  $t+2$ —via the cost price that will have to be paid for them at this moment—and so on.

In our opinion it is preferable to think that all processes—production, circulation and distribution—are taking place in time, of course, but they are happening also simultaneously and continuously. Therefore, when observing their evolution one can be interested in taking a picture of their situation at whatever moment  $t$ . The fact that the cost-price has to be taken as “given” when conceiving of and reckoning values, does not necessarily imply treating it as a “past” cost in historical terms, but past only in logical terms. What is crucial for the analysis is to *treat differently* the price of the inputs, which every single capitalist takes as given<sup>xlv</sup> because he feels it out of his control, and the profit included in the foreseen prices of the output—as a target associated with both the maximization of profits and at the same time the diminution of costs and prices to a minimum, which is the main arm in the free competition of capitals for achieving the former (see Shaikh, 1980).

This difference is crystal clear in Marx. First of all, “the definition of constant capital given above by no means excludes the possibility of a change of value in its elements (...) It is plain, however, that these changes of value are *independent* of the increment or surplus-value added to the value” (Marx, 1867, p. 135; emphasis added). It can be captured as well by the following simile:

“The circumstance, however, that retorts and other vessels are necessary to a chemical process, does not compel the chemist to notice them in the result of his analysis. If we look at the means of production, in their relation to the creation of value, and to the variation in the quantity of value, apart from anything else, they appear simply as the material in which labour-

power, the value-creator, incorporates itself. Neither the nature, nor the value of this material is of any importance” (Marx, 1867, p. 141).

The latter is just a consequence of the necessity of distinguishing the *labour* that the workers perform during the labour day, that creates new value, from the *work* that serves them to transfer the value of the means of production from the input to the output. Do not forget that Marx thought that his main contribution to economics (not only to economics) was the discovery of the “double character” of labour. And also, as Saad-Filho puts it, this distinction is just in accord with Marx’s “principle” that “profit is created in production, and that it depends primarily upon the *quantity* of labour power put in motion, rather than the value of the means of production (...) The ‘non-transformation of the inputs’ cannot be considered a defect. Rather, it is a *feature* of Marx’s method” (2002, pp. 153-4, 161).

Consequently, it should be said that, in real time, the only logical interpretation is to conceive of values and prices at any moment  $t$  as *a unique and single set of values and prices* although their evolution in time makes them to change continuously indeed<sup>xlvi</sup>. And, more precisely, our contention is that in the set of equations (1)-(3), the same fact that the TSS wants to apprehend by means of its “temporalist” equations, is taken into account by evaluating all the costs at prices  $m$ —prices that are (relatively) *passively* taken as given by capitalists—whereas at the same time treating outputs and profits as the result of the *active* struggle of all actors involved in production, the main variables that, as a living reality, can be significantly influenced by the effective behaviour of capitalists and workers, and at the same time cannot be quantified until both the production and the circulation process has been finished.

The new values or prices created in this way by the workers labour are for this reason “new” ( $mKR$ , where the profits included in  $R$  already reflect the main actual results of all those struggles), whereas the other values (prices) are “old” (and equal to  $m(A+B)$ ). Therefore, the vector of prices is one single vector, and this so regardless of it is computed at  $w$ ,  $p$  or  $m$  prices.

Let us look now more carefully at all this by means of Figure 2, where are represented two different things:

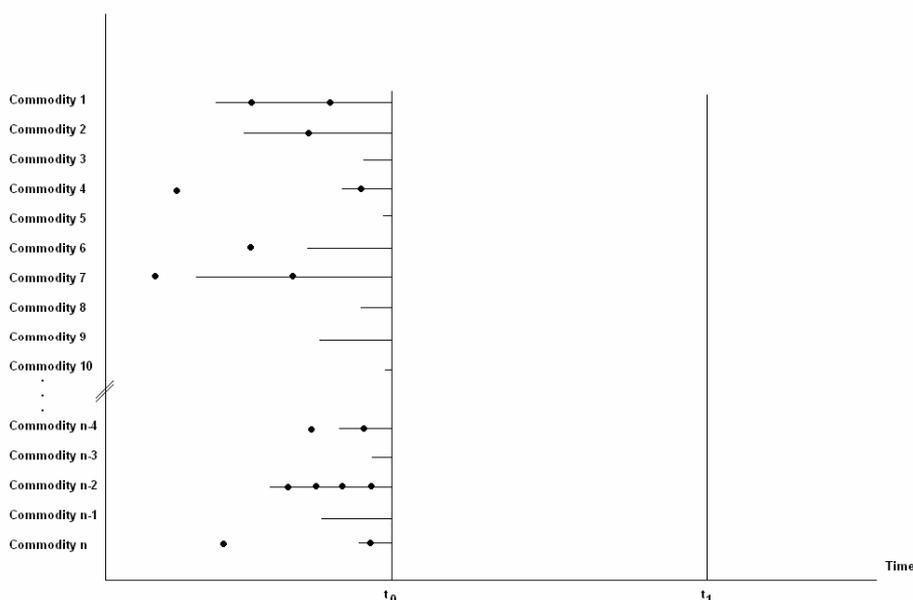


Figure 2: The inputs of commodity  $j$  and their values at time  $t_0$ .

1) On the one hand, commodities 1 to  $n$  are the means of production used as inputs in the production of commodity  $j$  whose single value we want to know<sup>xlvii</sup>. Every production process of the  $n$  means of production will in general be different in duration. Although all those processes are continuous—i.e. they are taking place continuously in a set of firms that “work” in the form of repeating them once and again and thus obtaining a certain mass or flow of commodities by period of time—we can represent the “unit” duration of each individual “production time” as the length of one of the segments ending exactly at day  $t_0$  in the figure (i.e. ending in the vertical line passing through  $t_0$ ), which is the moment at which we are interested in knowing the value of  $j$ .

2) On the other hand, the black points represent the moments at which at least one of the firms producing  $j$  has actually bought in the past (probably at different average prices if we suppose continuous technical change) a certain amount of those commodities as inputs. Note that not all commodities need to enter as inputs in a certain lapse of time (for instance, the one going from 0 to  $t_0$ ). Similarly, for the inputs which do actually enter in a lapse there is no need of being bought with the same frequency by all firms of sector  $j$ .

Now, one may legitimately ask what is the value of  $j$  (or  $l$ ,  $m$ ... or all of them) at  $t_0$  (or  $t_1$  or whatever moment). And since we can repeat the same question for all commodities, our task is to know the magnitudes of every element entering in the three vectors of value ( $w$ ,  $p$  or  $m$ ). But as we have just seen, this has nothing to do with the moment at which one single commodity is being bought, produced or sold by any one of the firms existing in the economy. As Figure 2 shows, the meaning of what Ramos calls moment  $t-1$  is perfectly ambiguous because there are in fact many different moments at which the different inputs are being purchased (in practice they are bought continuously), and the problem becomes greater if one thinks of the entire sector instead of the individual firm.

The relevant question for the LTV as conceived by Marx and as a useful tool for understanding reality is about the value of every *species* of commodity *now*. In principle, this “now” can be referred to any of the *moments* in time in real life, although it is true that if we are interested in reasonable studies on values throughout the whole economy it is more pragmatic to interpret those “moments” as lapses of time longer than one instant or even one day or week. For practical reasons, it sounds wiser for a majority of cases to conform oneself to thinking of values as average magnitudes referred to longer periods like one month or one year<sup>xlviii</sup>, not in daily terms, exactly like every macroeconomist or accountant use to think of the GDP of a country or the balance sheets of one firm. Moreover, this could also be the only procedure compatible with the fact that any theory of value does not intend to explain all values of all variants of every single commodity at every place and moment, but just the averages and the trends of the normal flow of every species of commodities.

## VII. A brief summary of the history of the debate

Having seen that the conventional equations of values and prices are mistaken in the general case, which place should we accord to them?

In effect, equations (1) to (3) are the correct ones because they correspond to the general case, the most complete level of analysis to which points the LTV. We know that Marx was unable to finish his ambitious plan of life-working and explicitly recognized not to have begun (or at least finished) the analysis of the world market, the state, etc. We guess that had he ended his plan, it would be easier for all to see that our equations are in accord with Marx’s conception. The main obstacle to seeing this at present is perhaps that our equations are valid in the most general case, and this is even more general than the model one can find, in finished state, in *Capital* (which is the model that guides us by the way). It regards a capitalist society where all actors are *always* present—not only, as in almost the entire *Capital*, the most important ones: capitalists and workers—and this case is more complete, more finished so to say, than the framework used by Marx to develop his main set of values and prices. His analysis of land, the state or the monopolies... is in this sense only a partial one, and even if his insights on them are

extremely insightful, he was unable to incorporate them fully into the analysis of a “general transformation procedure” reaching market prices too at the same level as production prices.

Notwithstanding this, it is necessary to admit that the usual equations that we have criticized along the paper should not be completely dropped. Instead, once put in the limited place that corresponds to them, they can still be useful. But their partial, not general, validity can be better appreciated by putting the analysis in relation to one of the three alternative scopes to which the LTV can be applied, which in turn helps to decide about the relative usefulness of each proposed set of equations.

I. Scope I. In the simplest context, corresponding *only* to what Marx developed in volume I, the focus is upon prices that are proportional to labour-values because there is *no production price yet* and direct prices suffice to understand why and how prices are only a name for values (labour), and profit for surplus-value. It would be thus possible in this context to write the same equations (in double form) used in the literature for values as direct prices:

$$(29) \quad w = w (A+B+B\rho)$$

$$(30) \quad w = wA + l$$

Equation (30) shows that values can be understood in effect as “vertically integrated labour coefficients”<sup>xlix</sup> ( $w = l \cdot (I-A)^{-1}$ ) and could also be directly obtained from an eigenequation completely similar to the more common one used for production prices (see below):

$$(31) \quad (1/\rho) \cdot w = w \cdot B'$$

[where  $B' = B \cdot (I-A-B)^{-1}$ , and  $w$  is the eigenvector associated with the maximum eigenvalue of matrix  $B'$ , i.e. the reciprocal of the rate of surplus-value,  $1/\rho$ ].

II. Scope II. In an *intermediate step*, it would be possible to define two types of prices instead of one. In this case we have both direct prices as well as production prices that appear in a context larger than step I, but still this scope is more reduced than the general one. This well-known pair of prices (the equations of the so-called “values” and “production prices”) *just* takes into consideration the *pure* competition between the many individual *industrial* capitals (i.e. competition inside the “productive” sector of the economy), and therefore are valid only in a theoretical context where no room is made for, among others, commercial and financial capital (although this is uncertain), private property of land, monopolies and the state. It is important to recall that it has been *only in this limited context* where has taken place almost the entire debate on the LTV and the TP for more than a century.

However, even in this limited context, the correct equations are not the usual ones—see the couples (35)-(36) and (37)-(38)—but instead the following:

$$(32) \quad w = p (A+B+B\rho)$$

$$(33) \quad p = p (A+B+Kr)$$

[where costs and stock are always evaluated at  $p$  prices even in defining  $w$  prices].

The first authors who have made this are Wolff, Roberts and Callari (1982), but this is also the position of Rodríguez-Herrera (1994) and, implicitly, of Itoh (1992) and Moseley (1993) too. However, Foley thinks that equation (32) “offer no economic motivation or interpretation of the ‘value’ coefficients”. And Mohun (2004) has similarly criticized this interpretation—that he calls “a single system ‘generalization’ of the NI”—by comparing it with the NI (see below), which he thinks preferable, because in the former “the different means of production comprising its elements will in general be produced in processes with different compositions of capital”, so that “at the prevailing value of money, the labour time equivalent of the money value of the means of production is *not* the aggregate embodied labour value of the means of production.” (2004, p. 79)<sup>l</sup>.

Of course, in this limited context, and in both this case (equations (32)-(33)) and the two following, it would be possible to “obtain”<sup>li</sup> the vector of production prices as the eigenvector

associated to the maximum eigenvalue (the reciprocal of the rate of profit:  $1/r$  in this case) of matrix  $K' = K(I-A-B)^{-1}$  since it is possible to write equation (33 = 36 = 38) as:

$$(34) \quad p \cdot (1/r) = pK(I-A-B)^{-1}$$

By contrast, a great majority of authors write the couple of equations wrongly even in this second, intermediate context, since they do not use the correct equation (32). They write the pair either as in (35)-(36)—in the way inaugurated by Bortkiewicz and followed by almost everybody except by the authors linked to the NI—:

$$(35) \quad w = w(A+B+Bp)$$

$$(36) \quad p = p(A+B+K)$$

or as in (37)-(38), like the NI does<sup>iii</sup>:

$$(37) \quad w = (wA+pB)+pBp$$

$$(38) \quad p = p(A+B+Kr)$$

Let us say that the NI was an unquestionable step forward. But, as has been remarked by Moseley, Rodríguez-Herrera and others, it can be interpreted also as an ambiguous middle point<sup>iiii</sup>. The fact that in this interpretation some inputs (the material inputs bought by constant capital) are measured at direct prices when defining  $w$  prices, while at the same time the “inputs” of the consumption of workers (paid with the income obtained from variable capital) are evaluated at  $p$  prices, seemed to many a kind a contradiction that opened the door for the search of alternatives. This gave way to the development of better approaches like the one represented by equations (32) and (33) already commented.

III. Scope III. But the fact that Marx was unable to end his planned work should not prevent others working in his wake to try to go beyond him. In the more general context, *all factors* disturbing the trend towards uniformity put in motion by inter-sectoral competition should be *fully* taken into account, completing in this way the task undertaken by Marx. Among these factors we find:

- 1) equalization of the rates of profit finds its limits due to the barriers raised by both the property of land and other natural resources, and the presence of monopolies in the context of a general framework of free competition of capitals;
- 2) differences in the individual conditions of production allow some units of capital to be more productive than others in each branch, which serves as a generalization of the concept of rent and at the same time makes necessary to take into account also individual prices ( $i$  in Table 1), especially the differences between “individual” prices and social prices;
- 3) the world market and the state are present as real facts and contribute with taxes, subsidies, tariffs and new practices of trade to new deviations of production prices from the average market prices; and so on...

It is thus in this most general context where the correct equations are, we insist, the trio of equations (1)-(3) of this paper, and therefore all other equations must in this case be interpreted as wrong, or at most as imperfect approximations to the correct ones. But we do not need to repeat all our arguments again.

## Conclusion

We would like to end with two notes on the history of the economic thought.

1. All those who have defended that the procedure used by Marx in dealing with the “transformation from values into prices” is correct are in general right, at least righter than their opponents. The main factor that lacked until now to clarify the errors committed by those critics of the LTV, in spite of their apparent superiority in terms of a more sophisticated use of algebra,

was to accept seriously on the part of the supporters of the LTV the challenge of going beyond the pure literality of *Capital*.

In dealing with values and prices Marx's words could not be, as happens necessarily to everybody writing a long work, found entirely and perfectly unambiguous. Although completely aware of the differences between the two, he tended to limit himself—when dealing with the Transformation issue—to think mostly in terms of production prices as the regulators of market prices, and in any case he worked and completed his treatment of prices of production in a more detailed way than what he did with actual market prices that “deviate” from them. However, to accuse Marx of over-emphasizing prices of production over market prices would not be just either, since he wanted above all to highlight the fact that everyday prices are *regulated* by others, and for this task he had to demonstrate that there are two regulators (one immediate:  $p$ , other mediate:  $w$ , but both different forms of labour), not just one. It is the same reason why he was correct as well in focusing on values and labour as the true determinants and regulators of production prices.

2. Nevertheless, from the new perspective developed in this paper, all subsequent participations in the secular debate about the TP from the point of view of the LTV could be seen as a succession of steps (not always following a straight line) directed towards the correct setting of the issue. We believe that the main obstacle all authors found to get earlier the same results defended here has been the almost universally oblivion of giving “real prices” the place they deserved in the analysis, whilst most of them confined themselves to discuss the problem only in the limited scope of the two “regulating prices”. Those authors were therefore unable to deal with the problem in a general framework, and the restricted character of the prevailing setting of the problem passed generally unnoticed, partly due in some cases to the habit of limiting the discussion of this and other “Marxian problems” just to the field in which one can feel accompanied by Marx's sentences.

### Appendix. More on the value of the means of production: ground-rent, interest and taxes

The issue of how to compute the value of constant capital raises two important challenges to the LTV. The first one has already partly been dealt with in the text: it relates to the treatment of historical costs versus replacement costs<sup>liv</sup> and especially the issue of how should the “past” or “given” *m* be interpreted. But there is a second dimension of the problem that has to do with the general relationship between the individual and the global or social points of views. Let us comment briefly one aspect of the first point before proceeding to the second one.

Suppose a firm buys an input at a higher price than the price paid by other firms in the same branch. This fact will certainly manifest itself in the form of a redistribution of profits between firms inside the sector, but this redistribution does not amount to a change in the price of the commodity whose value is determined by the *present* quantity of labour *socially* needed to reproduce it. The same is true when the inputs become (between the moment of purchase and their actual entry into the production process or their sale) more expensive, or cheaper, *for all* the firms of the branch at the same time—what is a hardly believably case by the way. Again in this case the effect would be a redistribution of profits, but now between different branches according to the evolution of the relative price of the respective commodities acting as inputs or outputs (at their turn conditioned by the evolution of technical change and the value composition of capital). But again there would be no change in the rule observed by Marx that it is the *present* social value (i.e. present, not past, quantities of labour) which counts.

When authors debate this issue they are in most cases dealing with the case of the individual producer operating in conditions of production that differ from the average of the branch. Let us think of a firm producing in inferior conditions. In this case, for a similar output this firm will actually spend more individual time of labour (direct and indirect labour) than the firms operating at the average or the best conditions in the branch, but the market will not validate its entire individual time (that includes both present and past time) as social labour and value (computed at present) and therefore its actual costs and/or profits will not appear entirely as a part of the present social average value in the branch’s commodity. Authors supporting the TSS approach would not deny this, and therefore it is difficult to understand why they stick to the idea that the entire actual expenses incurred by firms have to be part of the price of their output even when the price of an input has decreased by the time the values are been computed.

Finally, before dealing with the ground-rent, let us add that there is no need to let the time “be apparent” in our equations, so to say, in order to understand how values and surplus values are formed in temporal reality indeed. It suffices with observing that in every single day, or even in the shortest lapse of time one can imagine, all workers are truly working and thus the entire labour-power is performing labour and thus continuously creating new value<sup>lv</sup>. Therefore, it is perfectly possible to use the static tools of the prevailing approach for revealing the dynamic reality of values and surplus values that lies behind market prices. What of course does not amounts to saying that everybody using simultaneous static equations is in fact doing a dynamic analysis. Whether he uses or not a dynamic perspective will depend upon the set of theoretical assumptions he is making on the whole.

But there is a second crucial specific issue to be treated in this appendix: should we include among the inputs purchased by the capitalists those commodities that have a price but no value? First, should their prices count in the value of constant capital. And second, what about the rent paid by the capitalists out of the value added in their firms?

Of course we are referring to land and other natural resources but also to all products of labour that are not freely reproducible at present, like the works of dead artists<sup>lvii</sup>, rare old wines, etc. (see Ricardo, 1817, ch. 1). We could even extend the problem beyond this case and ask ourselves for the meaning of such things like the interests paid to banks or the taxes paid to the state. If we are coherent with our own equations (1) to (3), it is clear that we should compute rents, interests and taxes among the cost or value of constant and variable capitals as they are part of the price already paid by those inputs. But what to say about the ground-rent, interests and taxes paid by the firms *from their value added*, i.e. apart from those already included in their inputs?

Let us focus only on section VI of the third volume of *Capital*, since land can also be treated as “interest-bearing capital”<sup>lvii</sup>. Both cases could be dealt together, but note in passing that when authors like Samuelson, Sraffa and their followers—and, more surprisingly, Foley and Mohun too—all in the wake of Böhm-Bawerk, defend the idea that labour realized at different moments has to be “dated” since the older one has to be multiplied by a factor of interest ( $1 + i$ ) when compared with the more recent one, are supporting a theory that has nothing to do with the LTV against which Böhm-Bawerk’s theory was directed. Marx is crystal clear in this point, and the only thing we need to add now is that the reason for this is that the value of the inputs entering the cost-price has to be measured at  $m$  prices. These are prices that arise from real processes compatible with the fact that in a world of exploitation of labour, and therefore of profit of capital at a certain rate, the “simple” pass of time means an increase in capital in the form of money, and this money can always gain an interest at another rate,  $i$ , during the period in which it is out of the production process.

Marx is crystal clear on the question of time: “Hence, in determining the value of the yarn (...) all the special processes carried on at various times and in different places, which were necessary (...) may together be looked on as different and successive phases of one and the same process. The whole of the labour in the yarn is past labour; and it is a matter of no importance that the operations necessary for the production of its constituent elements were carried on at times which, referred to the present, are more remote than the final operation of spinning. If a definite quantity of labour, say thirty days, is requisite to build a house, the total amount of labour incorporated in it is not altered by the fact that the work of the last day is done twenty-nine days later than that of the first. Therefore the labour contained in the raw material and the instruments of labour can be treated just as if it were labour expended in an earlier stage of the spinning process, before the labour of actual spinning commenced” (Marx, 1867, p. 120).

It is also clear that, also when dealing with ground-rent, Marx acknowledges, as in many other places but not always, the differences between  $w$ ,  $p$  and  $m$ . For instance, he refers to the products “sold at their values (at a later stage of development they are sold at their prices of production) or at prices which are certain modifications of these values or prices of production determined by general laws” (1894, p. 435). More precisely, “it is in general in the form of the market-price, and, furthermore, in the form of the regulating market-price, or market-price of production that the nature of the value of commodities asserts itself” (p. 440).

Secondly, Marx writes that “rent, and therefore capitalised rent, the price of land, can enter as a determining factor into the price of agricultural products in only two cases”; but those cases are: “First, when as a consequence of the composition of agricultural capital—a capital which has nothing to do with the capital invested in purchasing land—the value of the products of the soil is higher than their price of production, and market conditions enable the landlord to realise this difference. Second, when there is a monopoly price” (p. 550). Therefore, this means that we are facing the general case, as agriculture was in Marx’s times, and still is, a sector with relatively low composition of capital<sup>lviii</sup>, and on the other hand the monopolist elements in this field—that he judges as more common in urban lands and mines (including oil) and linked to speculation—seem, contrarily to what happens in the industrial sector, to have increased (p. 529).

Of course, land has no value and therefore no price of production; however it does have an actual “price”, even if this price is for the same reason “irrational” and “nothing more than the capitalised rent” (p. 445). This is why we should apply to this case the general principle and use the *market* prices for all costs since this does not regard the process of creation of value: “The only requisite is that there be a sufficient supply to absorb the labour expended in the process of production. That supply once given, the material may rise or fall in value, or *even be, as land and the sea, without any value* in itself; but this will have no influence on the creation of value or on the variation in the quantity of value” (Marx, 1867, p. 141; emphasis added).

Now then one might be amazed by the literality of some sentences of Marx that seem problematic. For example, think of the two surprising and seemingly contradictory statements like the following: on the one hand the price of land (or the price of the natural waterfall, in the particular case he is considering at that point) “does enter into the individual cost-price of the manufacturer”, but on the other hand it “does not immediately enter into the production price of

the commodities” (ibid.). It seems like if he were saying that—in terms of our own notation—land should be a row in matrix  $A$ , but not in matrix  $K$  (and therefore neither in  $Kr$  nor in  $p$ ), but as we will see this is not the case.

Furthermore, he repeats later that there is a “conflict between the price of land as an element in the producers’ cost-price and no element in the price of production (even though the rent enters as a determining factor into the price of the agricultural product, the capitalised rent, which is advanced for 20 years or more, by no means enters as a determinant)” (Marx, 1894, p. 552). Which could appear furthermore reinforced by his characterization of the believing “that land itself possesses value and thus enters as capital into the price of production of the product, much as machines or raw materials” as a pure “illusion” (p. 550).

In the same vein, Marx repeats several times that the land price is not a true price “but capitalised and therefore anticipated rent”, and thus the “capital” that land represents for the landowner is not true capital<sup>lix</sup>. This false capital “forms neither a part of the fixed, nor of the circulating, capital employed here”; it “is neither investment nor working capital<sup>lx</sup>, any more than the capital which someone invests at the Stock Exchange in purchasing stocks or government securities, and which, for him, represents a personal investment of capital, is ‘invested’ in any branch of production” (pp. 549 and 554, note).

But note that what Marx is defending here is that because “landed property has nothing to do with the actual process of production” (p. 559), it is necessary to take off the rent of land from the overall surplus-value produced, thus being only the remaining part—i.e. the “profit of capital (profit of enterprise plus interest)” —what is redistributed among the producers, the true capitalists, via the price of production, as an “average profit” that has to be in proportion to the *real capital* invested (ibid.)<sup>lxi</sup>. The consistent procedure would therefore be to put aside the price of land and not to count it among the elements of the stock of constant capital at the time of computing the rate of profit. However, one could ask if, as the figure of the landowner has practically disappeared as an independent figure and, as pointed out by Marx himself, has melted into the figure of the capitalist, it might be more appropriate, at least as a practical matter<sup>lxii</sup>, to include the ground-rent into the sum of surplus-value that has to be distributed among capitalists as a whole (now become also the main landowners: they and their companies) and then to compare the aggregate with the sum of both the true (working) and the false (land) capital included in their balance sheets.

Lastly, regarding the “illusion” mentioned above, it is true that chapter 50 of *Capital* deals in effect with the “illusions created by competition”. But note that Marx wants to warn here against the danger that “appearances” and “experience” contribute to veiling or replacing scientific understanding. He calls attention against the possibility that remains unnoticed the error in this particular case, since

“the real movement would necessarily appear in distorted form; not as the splitting of a previously given magnitude of value into three parts which assume mutually independent forms of revenue, but, on the contrary, as the formation of this magnitude of value from the sum of the independent and separately determined, each by itself, constituent elements—wages, profit and ground-rent. This illusion would necessarily arise, because in the actual movement of individual capitals, and the commodities produced by them, not the value of commodities would appear to be a precondition of its splitting but, conversely, the components into which it is split function as a precondition of the value of the commodities. In the first place, we have seen that to every capitalist the cost-price of his commodities appears as a given magnitude and continually appears as such in the actual price of production.” (p. 592)

But it is crucial to understand which the purpose of this chapter is. Marx is criticizing here the wrong idea that there would be other elements apart from labour able to form value. He is showing why the economic agents have to be and are unable to distinguish between variable and constant capital, and why no difference is made by them between the factors that produce wealth (which are several) and the factor producing value which is only one (labour), and so on. But once understood this, the “appearances” he is criticizing in this chapter become the same kind of appearance as for example prices when compared with values. They are not only a

“necessary”<sup>lxiii</sup> appearance, but also as “real” as the essence they manifest, and thus they should be at that extent taken into account.

We should conclude then that this appearances do not eliminate the real fact that the capitalists count the ground-rent they pay as a *cost* and therefore that it should be applied to it the general rule: “the value of the means of production (...) enter into the production of his commodities as a given price magnitude” (p. 594), as for example in one of his preferred examples: “The values of the means of production, *i. e.*, the cotton and the spindle, which *values* are expressed in the *price* of twelve shillings, are therefore constituent parts of the *value* of the yarn, or, in other words, of the value of the product” (Marx, 1867, p. 120; emphasis added).

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<sup>i</sup> According to Marx, "aside from the confusion which the transformation of values into prices of production brings about, another arises due to the transformation of surplus-value into different, special, mutually independent forms of revenue applying to the various elements of production, i.e., into profit and rent" (Marx, 1894, p. 574).

<sup>ii</sup> Of course most authors have their own way of writing their equations that do not coincide exactly with the way they are expressed here. Think simply that a majority of them use models of pure circulating capital where there appears no stock of capital at all, contrarily to the  $K$  appearing in equations (1)-(3). In any case, it is always possible to say that ours would be the equations they would use in case they employed models including the stock of capital. But these differences do not seem to affect the main issue at stake here.

<sup>iii</sup> A partial and obscure exception is Guerrero (1999), where some of these ideas are presented in the framework of a different interpretation that the author no longer shares.

<sup>iv</sup> These prices should perhaps be called *actual* prices better than *market* prices since there are some occasions in which instances other than the market (for example the state) intervene in the fixing of prices (although it is true that even in that case market influences has to be taken into account, sometimes in a decisive way). However we will use the letter  $m$  (from "market") to design those prices, with the warning that we are not forgetting the extra-market factors just mentioned.

<sup>v</sup> By the way, it is worthy to clarify from now what is, in this field as in any other, the relationship of the author with Marx's ideas. He believes that: a) any theoretician must try to understand first of all the reality as well as possible; b) his understanding in this field has been indeed conditioned by Marx more than by any other author; c) his view is compatible with Marx's and is in many cases Marx's; d) however, in case that others believe that the latter is not true, or even in case that his understanding is actually incompatible with Marx's in certain respects, he still defends that it is his approach, not Marx's, the correct one to approach reality.

<sup>vi</sup> What does not mean indeed that all their users, among them the author of this paper in previous works, necessarily share Bortkiewicz's ideas.

<sup>vii</sup> In fact the influence of the social means of education and communication on the history and development of all kinds of ideas applies perfectly well inside the Marxian field of thinking too, as shows the frequently distorted state in which many of them have been transmitted from generation to generation of Marxists.

<sup>viii</sup> Let us see a few examples: in their New Palgrave's review, Hunt and Glick (1987, p. 689) say that getting all simultaneously is "impossible", Valle (1978) thinks that many people have "proved" the impossibility of the double [multiple] equality, and more recently Ian Wright (2006) has considered the opposite view a "Theorem". This believing has led to many authors to believe that Marx's view of the TP is either an error or an incomplete solution. In fact, the characterization of Marx's position has received almost as many adjectives as authors. Most of them follow Sweezy (1942) in considering Marx's "solution" either a "mistake", or a "failure" (Szumski, 1989), an "error" (Fine and Harris, 1979, p. 31; Lipietz, 1999, p. 1163) or the result of "a number of errors" (Mongiovi, 2002) that would make the whole theory "inconsistent" or "contradictory" (Bellofiore, 1997, p. 9). Others, following Mühlport (see Jorland, 1995, p. 263), soften their critique and just see Marx's solution as either a simple "defect" that makes necessary to "complement" it (Foley, 1986); or a solution that is "incomplete" (Harvey, 1982, p. 64; Cockshott and Cottrell, 1995) or "insufficient" (Valle, 1997, p. 136), but in any case not "rigorous" or "not satisfactory" (Duménil and Foley, 2006). All this critiques have a long tradition. In fact, before Sweezy, many Marxists from Bernstein to Moszkowska (1929) and Shibata (1933) did the same (see the extensive reviews of this literature, almost unknown in English, made by Jorland, 1995, or Rodríguez-Herrera, 1994; the latter is by the way an important book where a view similar to that of this paper is defended and in particular the point that there is no mistake or incompleteness in Marx's solution, but instead in his critics).

<sup>ix</sup> Those studies clearly reproduce Marx's practical thinking of the problem: "If one examines price lists over a more or less long period of time, and if one disregards those cases in which the actual value of commodities is altered by a change in the productivity of labour, and likewise those cases in which the process of production has been disturbed by natural or social accidents, one will be surprised, in the first place, by the relatively narrow limits of the deviations, and, secondly, by the regularity of their mutual compensation" (Marx, 1894, p. 586). It is a pity that some supporters of the LTV seem to think, mistakenly in our opinion, that there are insurmountable statistical problems that obstruct the correct calculations of these deviations and correlations (see for example the debate on this issue among Kliman 2002 and Díaz-Calleja and Osuna 2005).

<sup>x</sup> We believe that there are other possibilities still waiting for being developed in the field of empirical studies. Think for example of the study of the evolution in time for a given period of both the disaggregated prices of a bulk of important and well known commodities, at either national or international levels, and their regulating labour-values.

<sup>xi</sup> In fact, some authors, not only the TSS, seem to criticize the mere use of some mathematic tools like matrix algebra, calling its users “matricialists”, “algebrists”, and so on. The relationship between theory and mathematics regarding the LTV and the TP seems to be in this field as problematic as in other fields of economics. It is of course a frequent error to fall under the hypnosis produced by the beauty and usefulness of mathematics, but it is not a good remedy trying to avoid this error by forbidding oneself its use. It is not worthless to emphasize that the problem is not mathematics, algebra or whatever else, but instead the use of the material assumptions that define every theoretical model. Everybody should use the formal tools they want, but only in order to deal with reasonable and realistic assumptions. It is clear that one can use matrix algebra without necessarily sharing the ideas defended by other users, just like one can use the equations or the entire graphical equipment used in the conventional teaching of Microeconomics courses without necessarily sharing the political views of other colleagues.

<sup>xii</sup> We agree with Foley who writes that the LTV for Marx “was a theory of exploitation and of money”, whereas most of his critics “manage to ignore completely the monetary aspect of Marx’s labor theory of value” (2000, pp. 7, 18). Foley thinks that he and Duménil’s “reconstruction” of the LTV has been precisely made by “emphasizing the relation between money and labor time” (p. 19).

<sup>xiii</sup> Bródy has pointed to the physical input-output used by Marx in the *Grundrisse*—the first one registered in “economic science”, he says—specifying that it served him to “analyze product flows on one hand and value flows from the dual viewpoint” (Bródy, 1970, p. 33).

<sup>xiv</sup> See for instance Leontief (1966). It is not only that all the data of his input-output tables are computed “in billions of US dollars” (p. 65), or that there is a row for “taxes” or “government” (pp. 66, 72, 79-80), or that he is continuously talking of “purchases” and “sales” and market “transactions” (pp. 73, 76), but above all that he is perfectly aware of “the variations experimented by the structure of inputs as a result of the variations in prices and in the technology” (p. 71; all pages cited from the Spanish version and translated back by DG; emphasis added).

<sup>xv</sup> Note that “the average price of labour, i.e. the value of labour-power, is determined by the production price of the necessary means of subsistence” (Marx, 1894, p. 591). For an interesting discussion of the value and price of the labour power, see Gouverneur (2005). This issue has been extensively discussed by the NI in their point about the necessity of using what we have called *p* prices also in the definition of *w* prices. However, Fine, Lapavitsas and Saad put forward some criticisms of the NI because, according to them, “no direct account is taken of social and historical elements in the value of labor power, other than the shifting balance of forces between capital and labor; for example, how does the money wage relate to the economic and social reproduction of the workforce, of which the customary standard of living is one component? Second, the value represented by wages bears no relation to the value of the commodities consumed, given that prices and values diverge from one another.” (2004, p. 9). This seems to be an unjust critique of the NI, since: 1) the historical elements are already present in the level of wages and prices actually defined at present; and 2) the link between wages and the values of the commodities purchased with them is the same than that existing between value and market price in general, as makes clear the quotation of Marx above. These authors say that “the value of labor power is neither a quantity of money nor goods but a quantity of value” and is “determined at the aggregate level through the exchange between capital and labor as a whole (i.e., as social classes), prior to the process of production. This is because at the most abstract level, advancing the value of labor power is a *precondition* for the production and realization of surplus value and, subsequently, the performance of labor and exploitation in production” (ibid., p. 10; emphasis added). It is precisely because those “inputs” are a “precondition” for producing value and surplus-value that they should be treated exactly like the inputs entering directly into the production process, and thus computed at their market price. On the other hand, the fact that the value of labour force is determined in this way does not mean, contrarily to what these authors defend, that it is fixed “independently of the contradictory tendencies associated with the accumulation of capital” (p. 11).

<sup>xvi</sup> It is crucial to see that wages can and should always be interpreted as being at the “subsistence” level, not as determined by any physical quantity, but as far as the wage workers cannot actually become members of another class, i.e. transform themselves into capitalists or self-employed workers. The real processes of proletarianization observed in all countries (see Guerrero, 2006) is the proof that wages are actually at this subsistence level, since it is greater the number of the members of the other classes (included their descendants) who become wage workers than that of the wage workers who move in the opposite direction. For other arguments in favour of this thesis, see Ong (1980).

<sup>xvii</sup> As said above, since we intend to focus on the general case, attention is paid only to a system of equations where fixed capital is present, not only circulating capital, even if no essential conclusion would change if we used instead the more usual models where fixed capital is absent. Bródy (1970) and Shaikh (1998) follow the same criterion, and see Duménil (1975, pp. 204, 208) for an explanation of why there is no need to account for a stock of variable capital in the overall stock of capital because it is already included in the value of the elements constituting the material “body” of constant capital.

<sup>xviii</sup> They agree also in that “there is never any state of equilibrium in which market prices ‘converge’ to prices of production” (Shaikh, 1992, p. 77). Another way to put it is by saying that all values and prices in part C of the table are concepts, but are concepts full of meaning, not just the result of simple formal calculations. See Shaikh (1977) on the difference between “calculation” and “conception” that can be found in Marx’s LTV, whereas in the neo-Ricardians one can find only the former.

<sup>xix</sup> Marx writes in *Capital*: “Money as a measure of value is the phenomenal form that must of necessity be assumed by that measure of value which is immanent in commodities, labour-time (...) Hence the expression of the equivalence of a commodity with the sum of money constituting its price, is a *tautology*, just as in general the expression of the relative value of a commodity is a statement of the equivalence of two commodities” (Marx, 1867, pp. 51, 54; emphasis added). In their criticism of the New Interpretation (NI) of the TP, Fine, Lapavitsas and Saad-Filho (2004) strangely follow Mongiovi, a critic of the LTV, in the interpretation of this “tautology” as a kind of error. They are right in that “equation (3)” of the NI [i.e.  $Rm = L$ , where  $R$  is “total net revenue”,  $m$  is the “labor expression of money”, and  $L$  is the “total amount of living labour”] should not be “taken by the NI to be the analogue of Marx’s proposition that total value equals total price” (2004, p. 5), which is of course a “vertical” movement. However, these authors err when they contradict Duménil and Lévy’s statement that “actually, the relationship between values and prices is fully independent from the fact that profit rates are equalized”, meaning that there is a horizontal movement which is independent of the vertical one.

<sup>xx</sup> He mentions de Brunhoff (1967) as the origin of this interpretation and reminds us that “Bortkiewicz and his successors neglect that money is a commodity and reduce it, as a measure of value, to a simple unity of account, exactly like in the neoclassical tradition” (1994, p. 27).

<sup>xxi</sup> Of course, the magnitude of  $\pi$ , is not the same as  $\pi_2$ , but the fact that they are respectively, let us say, 30 euro or 60 euro per hour of labour does not make any difference as regards the problem involved. So we will simply speak of  $\pi$  from now on.

<sup>xxii</sup> As puts it Foley, “Marx constantly uses this conception to move back and forth between money and labor accounts” (2000, p. 9). However, Shaikh (1992) has opposed the idea that the passage from B to A is as proper as passing from A to B. He criticized Itoh for this, and the same do Shaikh and Tonak (1994) regarding the New Interpretation: “As just defined, the value of money (...) is the living labor commanded *in exchange* by the net product. This means that the value of labor power (...) is the living labor commanded by the money wage bill of productive workers, and that surplus value (...) is simply the living labor commanded by the existing mass of profit. Marx argued that price and profits were monetary forms of value and surplus value. The new approach abandons this altogether by defining surplus value to be a form of profit! The whole relation between surplus value and profit is turned on its head. Moreover, this approach does not even have the virtue of being new, since it is really nothing more than Adam Smith’s second definition of labor value as living labor commanded by price. Ricardo and Marx decisively rejected this approach, with good reason” (1994, p. 179). However, Foley seems to be right in his response: “It is true that the New Interpretation identifies the phenomenal forms of price to the categories of the labor theory of value, but it is hard to see why this turns the relation on its head” (2000, p. 26).

<sup>xxiii</sup> Foley agrees with Marx and writes that the “monetary expression of labor” allows to “translate flows of money in real-world capitalist accounts into flows of labor-time and vice versa” (2000, p. 22). As Shaikh before him, Saad-Filho also criticizes this conception of the relationship value-price as “simply a circulation-based view of price” that limits itself to “the appearance” and “fails to give analytical priority to conceptually more fundamental processes such as the performance of labour in production” (1996, p. 128).

<sup>xxiv</sup> Note that the  $m$  have to be understood in the statistical sense, i.e. as average magnitudes in time and space, because it is possible for any commodity that every actual day-to-day transactions is made at prices that all differ from the average  $m$ . The fact that  $p$  prices are averages of the  $m$  in another sense, should not veil the understanding that in practice the  $m$  that can be known as data at the macro level are averages of the actual  $m$ . Of course, the same can be said of all categories in the table, which means that in it are represented their annual (or other periods of time) averages, not their day-to-day magnitudes.

<sup>xxv</sup> On this topic see also Shaikh (1984), who adds that the empirical measures seem to support the idea that actual prices are more closely approximated by direct prices than by production prices, which is by the way the same conclusion attained by Farjoun and Machover (1989), Cockshott and Cottrell (1995) and Zacharias (2006).

<sup>xxvi</sup> Other interpretations from the LTV are possible, following either Shaikh (1977), who interprets the TP as just a trip inside column B (from 2' to 3'), or Duménil (1983) who prefers to think of it as a passage inner to column A (between 2 and 3). Note that both interpretations can be accommodated inside the conception developed in this paper.

<sup>xxvii</sup> It is usual to speak only of two equalities, although in some cases the list is made longer. For instance, Foley mentions 4: 1) the equalization of the individual rates of profit; and the conservation of 2) surplus-value and variable capital; 3) constant capital; 4) and the same rate of profit in price as in value (1983, p. 607).

<sup>xxviii</sup> With the sole exception of Wolff *et al.* (1982) and Rodríguez-Herrera (1994), who gets some similar results but cannot go beyond this due to the absence in their equations of prices  $m$ .

<sup>xxix</sup> “Duménil and Lévy both pointed out the quantitative incongruity between the embodied labour coefficient accounts rate of profit and the rate of profit at market prices”. (Foley 2000, p. 24)

<sup>xxx</sup> Fred Moseley has paid due attention to this important idea of Marx (see Moseley 2000). What matters in the first place for the LTV is to understand the process by which capital as a sum of money-value can and in fact grows in time, and why this is not possible unless wage-labour is continuously exploited by capital at a certain rate. This is why among other things we must start for analysing values from a sum of money and interpret the inputs used in production as the purchases that the capitalist have to do in the pursuit of their main aim: the creation of value and surplus-value.

<sup>xxxi</sup> Methods for this are multiple and can be grouped in what Marx calls the extraction of “absolute” and “relative” surplus-value. As a matter of fact, each capitalist must achieve the extension of the labour day of all their workers as much as possible, and this is always possible although it will find for sure the opposition of workers. “This metamorphosis, this conversion of money into capital, takes place both within the sphere of circulation and also outside it; within the circulation, because conditioned by the purchase of the labour-power in the market; outside the circulation, because what is done within it is only a stepping-stone to the production of surplus-value, a process which is entirely confined to the sphere of production” (Marx, 1867, p. 124).

<sup>xxxii</sup> And also other participants in the struggle for a portion as high as possible of the common loot seized from workers (like landowners, the state, etc.), but they can be put aside in a first analytical moment.

<sup>xxxiii</sup> See on this Guerrero (2003).

<sup>xxxiv</sup> When we say that prices are “given”, i.e. that firms are price-takers, not price-determinants, we do not mean that we are defending the perfect competition model. On the contrary, competition must be understood as the free war of competition among all capitals (see on this Shaikh 1980 for more details on this conception, which is Marx’s), which clearly differs from both the perfect competition model and the monopoly model.

<sup>xxxv</sup> In fact, since all firms focus upon their own individual conditions of production—in particular trying to use technical methods permitting to obtain costs and prices as low as possible, which is the best arm in both battlefields—this struggle ends changing slowly and indirectly the general market conditions. However, big and quick changes in the individual conditions usually influence social conditions little and in a slow manner.

<sup>xxxvi</sup> By the way, no neoclassical or rraffian economist worry about the necessity of transforming  $p$  prices into market prices ( $m$ ), and this would be for many a good guide for doing the same in the passage from  $w$  to  $p$ , which is precisely what we will do below but due to more serious, theoretical reasons.

<sup>xxxvii</sup> In the sense that if the actual market price in a sector exceeds permanently its production price and therefore this sector is accruing a higher rate of profit than the average, the same sector will suffer an entry of new capital into it at a higher rate than the average, its supply will thus grow also at a higher rate and at the end its market price will come back to the level defined by its production price. This is just the expression of the concept of the Invisible Hand in Smith.

<sup>xxxviii</sup> To which it could be the way be added another equation for the “individual” values, as below, where the primes show that in this case we would not have  $n$  sectors but, supposing that there are the same number of firms in each sector ( $j$  firms), we would have  $n \cdot j$  equations for the individual values,  $i$ , and therefore matrices  $A$ ,  $B$ ,  $K$  and  $R$  would be now of dimension  $(n \cdot j \times n \cdot j)$ —that is why we write a prime—and vectors  $m'$  and  $i'$  of dimension  $(1 \times n \cdot j)$ :

$$i' = m' (A'+B'+K'R')$$

But note that in  $m'$  there are only one price for each type of input (i.e. a maximum total of  $n$  prices, each repeated  $j$  times), regardless of it coming from one or another firm of that sector.

<sup>xxxix</sup> When we say “actually expended” we mean spent at actual, market prices ( $m$ ), not at theoretical prices like  $w$  or  $p$ . But, as will become clear below, we are not proposing, contrarily to the TSS, to take historical costs instead of replacement costs as the value of constant capital, i.e. prices not at present ( $t$ ) but at the moment of the purchase ( $t-1$ ) (regardless of which prices are used,  $p$  prices as TSS do, or  $m$  prices as we propose).

<sup>xl</sup> Marx writes on this: “Since one portion of the value, as well as of price of production, is an actually given constant, namely the cost-price, representing the capital= $k$  used up in production, their difference consists in the other, the variable portion, the surplus-value, which equals  $p$ , the profit, in the price of production” (Marx, 1894, p. 521).

<sup>xli</sup> Note that three different figures could be derived from Figure 1: 1) one relating quantities and costs; 2) other relating costs and time; 3) and a third relating time and quantities (or scale). In Figure 1 we can see that costs are decreasing in time but at the same increasing as a function of  $Q$  (beyond the optimum of exploitation), and the scale of the firm is decreasing along the time too (which is contrary to what happens in practice more often).

<sup>xlii</sup> This author has been very close to the perspective used in this paper (he is in fact the coauthor of one of the chapters of the important book by Rodríguez-Herrera, 1994) but then departed from it in direction to the TSS approach.

<sup>xliii</sup> On the contrary, Moseley seems to be thinking instead in logical, not chronological terms: “Marx’s analysis of the sphere of circulation provides the logical presuppositions (the ‘givens’) for his analysis of the second phase of the circulation of capital in the sphere of production. Again, the Sraffian interpretation of Marx’s theory completely ignores this initial phase of the circulation of capital in the sphere of circulation, and implicitly assumes that capital first appears, not in circulation, but in production, as the physical inputs to production. This is clearly not Marx’s logical method in the first three parts of Volume 1. The initial quantities of money capital that provide the givens in Marx’s theory of surplus-value come from circulation, not from production” (2000, p. 292).

<sup>xliiv</sup> Note that Duménil and Foley (2006) call now their NI “the Single-System Labour Theory of Value (SS-LTV) interpretation”.

<sup>xli v</sup> And “given” also in a second sense: that cost-price has to be measured at the average, not the individual level, because the market-price of the inputs “is not determined by the individual cost-price of every single industrial producer, but by the average cost-price of the commodity under average conditions of capital in the entire sphere of production” (Marx, 1894, p. 440).

<sup>xli vi</sup> Like production, thought happens in time too. And if one of the results of present thought is the actual calculation of the set of values at present, any real agent can take these present data as one of the conditions of their future action. For example, the capitalists could take them as showing the conditions they have to take into account for better performing the process they begin now in order to get maximum profits tomorrow—which of course will be computed as part of tomorrow prices. This perspective does not contradict the necessarily dynamic perspective that should be adopted if one wants to place oneself in Marx’s trail. Duménil and Foley (2006) are right when they write that the “evaluation at ‘replacement costs’, however, does not imply that the economy is necessarily in a stationary state as the TSS critique has claimed”.

<sup>xli vii</sup> This means that we should repeat the same type of figure for all commodities different from  $j$ , because every commodity is different from any else, and therefore we would have to deal with as many different figures as commodities exist.

<sup>xli viii</sup> Among other reasons, and as already mentioned, because the market-price of the inputs “is not determined by the individual cost-price of every single industrial producer, but by the average cost-price of the commodity under average conditions of capital in the entire sphere of production” (Marx, 1894, p. 440). This is not indeed but an aspect of the general method used by Marx—contrarily to the methodological individualism—according to which, as highlighted by Moseley, “aggregate magnitudes are determined prior to and independent of individual magnitudes. Individual magnitudes are then determined at a later stage of the analysis, with the predetermined aggregate magnitudes taken as given” (Moseley, 2000, p. 285; see also Foley, 1982, p. 37).

<sup>xli ix</sup> Foley calls them “embodied labor coefficients” (2000, p. 22).

<sup>i</sup> In Mohun’s view, there are two main criticisms that can be made to this “generalised” single system. First of all, equation (32)—that he writes as (in our terms) “ $w_H = (p/\pi)A + l'$ ”—“sever[s] any linkage between (living and dead) labour embodied on the one hand and ‘value’ on the other save through the value of money” (2004, p. 81). And secondly, he agrees with Foley (2000, p. 34) in that this interpretation leads to the calculation of the “value of money as the ratio of the value to the price of gross output” as “a vector divided by a scalar, rendering the concept incoherent”, since “fixed capital is a stock of means of production produced at different times using different prevailing technologies” (ibid.). But Mohun’s arguments are not valid. First of all, if his first argument were valid—i.e. that using  $p$  instead of  $w_H$  in the right side of the equation of direct prices means that the linkage is severed between values and labour—he should conclude that is also “severed” any link between production prices and labour in the general case. And secondly, the value of the output is, contrarily to what he thinks, a scalar too, not a vector, since for the value of the means of production it does not matter that they “are produced at different times using different prevailing technologies”, but *only* that all of them need to be reproduced *at present* at the prevailing technologies.

<sup>ii</sup> “To obtain” means here “to calculate”. But, as we wrote above, the *capitalist rate of profit as a real fact* can neither exist nor be conceived of nor be calculated without the existence of actual processes of labour, and therefore without the existence of real labour relations that make possible the production and circulation of values. This implies that even if equation (33) were the correct one, this would not signify that the rate of profit and the prices *are* independent of values.

<sup>iii</sup> Duménil and Foley (2006) do not explicitly write this equation of values as the sum of both types of costs appearing explicitly in the form  $(wA + pB)$  plus the profit, as in (37). They write it only in the form “ $w_H = w_H A + l'$ ”, and define variable capital as a fraction of living labour (using the rate of surplus-value), but their results only obtain if they conceive of the latter as the same amount as  $pB$ . Put it in their own terms, their  $v$  (defined as  $1/(1+p)$ , where  $p$  is the rate of surplus-value) coincides with their  $w$  (the wage share in the value added, as measured at money terms). Or, as Foley puts it,

“the ‘value of labor-power should be measured as the ratio of the money wage to the monetary expression of labor, not as the labor embodied in the commodities workers consume” (2000, p. 25). However, Foley feels uncomfortable with this conception and, after being criticized by Saad-Filho (1986), claims “for a concept of the value of labor-power independent of the *ex-post* realized wage share” (p. 30; see Fine, Lapavistas and Saad-Filho, 2004). Finally, Duménil and Lévy put it in another equivalent way: “This is equivalent to saying that the rate of exploitation must be determined in nominal terms” (1993, p. 363).

<sup>liii</sup> Moseley argues that “there is a key methodological inconsistency in Foley’s interpretation between the determination of constant capital and the determination of variable capital. Variable capital is taken as given in money terms, but constant capital is derived from given physical quantities. Foley does not provide a rationale for this inconsistent treatment of constant capital and variable capital” (2000, p. 308). Furthermore, “Foley’s inconsistent treatment of constant capital and variable capital leads him to the following erroneous conclusions regarding Marx’s theory of prices of production (...) (1) Marx made a partial error in his determination of prices of production in Volume 3 (...) (2) The total price of commodities also changes from Volume 1 to Volume 3, so that the total price is no longer equal to the total value of commodities. (3) The rate of profit also changes from Volume 1 to Volume 3, i.e. the ‘price’ rate of profit is not equal to the ‘value’ rate of profit.” (p. 310)

<sup>liiii</sup> The point of view that favours historical costs is a strange conception, as Marx clearly points out that “if the time socially necessary for the production of any commodity alters (...) *all* previously existing commodities of the same class are affected, because they are, as it were, only individuals of the species, and their value at any given time is measured by the labour socially necessary, *i.e.*, by the labour necessary for their production under the then existing social conditions” (Marx, 1867, p. 136; emphasis added). Ramos accepts as correct Marx’s idea but thinks that this is true only regarding *the stock* of commodities that still exist at present, whereas it does not hold in the case of the commodities already disappeared as a result of their consumption in the production process they entered. Consequently, in his opinion the cost-price has to be computed at prices actually paid in a previous moment of time, even if the present price of the same commodities has become lower.

<sup>liiii</sup> It is this daily mass or flow of new labour together with the labour that the workers transfer this day from the means of production to the output that accounts for the entire mass of value of the commodities created during the day. Likewise, it is the difference between this amount of daily new labour and the labour needed to reproduce the daily subsistence of that labour force what explains the amount of surplus value produced during this day.

<sup>liiii</sup> In this case, by contrary, a certain quantity of value is present, but only as far as the making of the work of art consumed a certain quantity of the artist’s *wage* labour and so a fraction of social labour in a capitalist context. But the price of these works can deviate from their value in such an enormous measure that we should treat them as regulated only by demand, as in the case of land and other natural resources.

<sup>liiii</sup> The basis for this is clear. For instance Marx writes that for the buyer of land the money spent in buying it “can never again function as such, no more than any other money which he has definitely paid out. It figures in his accounts as interest-bearing capital, because he considers the income, received as rent from the land or as interest on state indebtedness, as interest on the money which the purchase of the claim to this revenue has cost him.” (Marx, 1894, p. 550)

<sup>liiii</sup> “It is possible for agricultural products to be sold above their price of production and below their value, while, on the other hand, many industrial products yield the price of production only because they are sold above their value” (p. 518).

<sup>liiii</sup> “The capital invested by the landowner himself in purchasing the land constitutes indeed an interest-bearing investment of capital for him, but has absolutely nothing to do with capital invested in agriculture itself” (p. 549).

<sup>liiii</sup> The buyer of land “now no longer has capital, but in its stead a piece of land” (p. 550); it is true that it is “capital *in itself*, just as any value sum is capital in itself, potential capital, on the basis of the capitalist mode of production (...) is a sum of money [and] this is capital in itself, because it can be converted into capital” (p. 549).

<sup>liiii</sup> Foley is in accord with this when he writes that “if the are taxes, payments to unproductive labor, rents, or interest payments, the tendency may be to equalize net profits after deducting these items” (1982, p. 46).

<sup>liiii</sup> At least, inasmuch as it is not possible to dispose of reliable data of a separate accounting of all rents in the economy and particularly those allowing to separate the rents from the pure profits in the cash-flows of the firms that own their pieces of land.

<sup>liiii</sup> But which is the secret to understanding this, according to Marx? Why are those appearances necessary? “The secret wherefore these products of the splitting of commodity-value constantly appear as prerequisites for the formation of value itself is simply this, that the capitalist mode of production, like any other, does not merely constantly reproduce the material product, but also the social and economic relations, the characteristic economic forms of its creation. Its result, therefore, appears just as constantly presupposed by it, as its presuppositions appear as its results. And it is this continual reproduction of the same relations which the individual capitalist anticipates as self-evident, as an indubitable fact” (pp. 593-4).